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Research Article

THE ENERGY EFFICIENT CONFIGURATIONS OF NATURAL CONVECTION HEAT TRANSFER IN SQUARE ENCLOSURES WITH HEATLINES

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ABSTRACT

A steady state natural convection heat transfer is numerically investigated in a two-dimensional square enclosure. One of the side walls of the enclosure is brought to high temperature and the upper wall is insulated. The other walls are partially insulated and the remaining parts of those walls are kept cold in accordance with the area of cold and hot surfaces are equal. The solution is performed for Rayleigh number values between 10^4 and 10^6, and for a Prandtl number value of 0.71. The obtained results are presented as local and average Nusselt numbers, streamlines, isotherms and heatlines. It is found that cold and insulated wall configurations have noticeable effects on the flow temperature fields and resulting heat transfer.
Keywords: Natural convection heat transfer, square enclosure, discrete heat sink, heatline.

Abbreviations

A = adiabatic
C = cold

Nomenclature

g = gravitational acceleration
H = height of the enclosure
Nu = local Nusselt number
$\overline{Nu}$ = mean Nusselt number
p, P = pressure, non dimensional pressure
Pr = Prandtl number, ν / α
Ra = Rayleigh number, $g\beta(T_h - T_c)H^3/(\nu\alpha)$
T = temperature
u, U = horizontal velocity component, non dimensional horizontal velocity component
v, V = vertical velocity component, non dimensional vertical velocity component
x, X = horizontal coordinate, non dimensional horizontal coordinate
y, Y = vertical coordinate, non dimensional vertical coordinate

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Greek Letters

α = fluid thermal diffusivity
β = fluid volumetric expansion coefficient
ρ = fluid density
ν = fluid kinematic viscosity
θ = non dimensional temperature
Π = heat function

Subscripts

c = cold surface
h = hot surface
l = left surface

1. INTRODUCTION

Natural convection heat transfer has a wide application field in the engineering practice such as electronic cooling devices, thermal comfort related configurations, distillation system mechanisms etc... Natural convection studies on the computational base have been increased with the increasing computational speed and data storage capacity of computers. Remarkable amount of square enclosure problem has been studied among the reported works on natural convection. Those studies are focusing on to increase or to decrease heat transfer within the enclosure in order to create energy efficient configurations. For the present study, it is intended to suggest a better arranged boundary condition to augment natural convection as an alternative to the existing boundary condition arrangements.

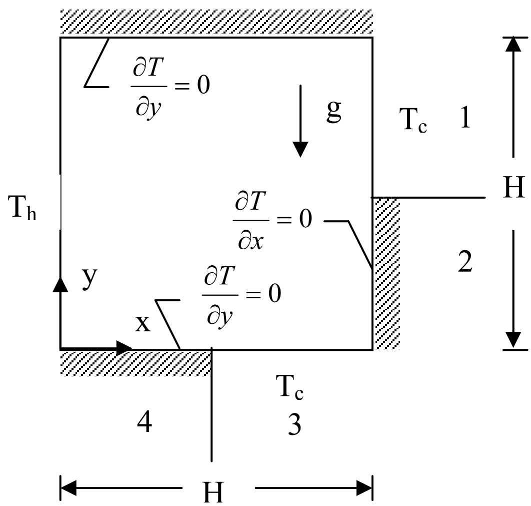
Aydin and Yang (2000) performed a numerical study for natural convection flow between Rayleigh number interval of 10^3-10^6 in a square enclosure. The square is subjected to localized heating condition at the base, cooled from the sides and insulated at top. Fusegi et.al. (1992) investigated the aspect ratio effect on natural convection in rectangular enclosures for different Rayleigh numbers. They prescribed the vertical surfaces hot and cold temperatures while the horizontal surfaces are kept insulated. Aydin et.al. (1999) studied the inclination angle effect on natural convection heat transfer in an inclined enclosure having hot surface at the left wall and cold surface at the top for different inclination angle. A square enclosure heated from bottom, insulated from and linearly heated by the sides is studied by Sathiyamoorthy et.al.(2007). A rectangular enclosure heated from lower half of left vertical wall and cooled from upper half of right vertical wall is studied by Alam et. al. (2012). Bilgen and Yedder (2007) analyzed natural convection in rectangular enclosure for different aspect ratios and Rayleigh numbers for the case of sinusoidal heating at one vertical wall and insulated other walls. Basak et. al. (2006) investigated heat transfer in a square enclosure for different Rayleigh numbers under uniformly/non-uniformly heated bottom wall, cooled vertical walls and insulated upper wall conditions. Natural convection heat transfer in an enclosure having equally distributed heat sources at the bottom is investigated by Bazylak et. al. (2006). Radhwan and Zaki (2000) analyzed the natural convection heat transfer in a one side partially heated and the opposite side cooled square. They performed the analysis for different lengths and positions of heaters. Djoubeir et al. (2014) carried out a SIMPLE based numerical solution procedure to search the effect of different heating parts on transient natural convection. A study on heat transfer in square enclosure by Namprai and Witayangkurn (2012) deals with the effect of heat source/heat sink located at the opposite vertical walls. Finite element method is used for the study and the results are presented in terms of isotherms, streamlines and heatlines for different positions of the source/sink couples. Square enclosure heated from bottom and one side is analyzed by Chen and Chen (2007). Natural convection heat transfer in an inclined square enclosure heated from two adjacent walls and cooled from the opposites is analyzed by Cianfrini at. al (2005). The SIMPLE procedure is u sed for the solution and the analysis is performed for Rayleigh number range of

10^4-10^6 and inclination angle range of 0°-360°. Based on Bejan's heatline concept (1983) Basak and Roy (2008) performed a natural convection study for square enclosure insulated at upper wall and heated for three different cases. Deng (2008) performed a numerical study on square cavity natural convection problem with two or three heat source-sink couples on its sides. He found that formation of the created swirl zones are related with numbers of the source-heat couples.

Natural convection in two dimensional square enclosure with insulated from horizontal walls and heated one of the vertical wall is investigated by Davis (1983) for Grashoff number interval of 10^3-10^6 . Ostrach (1988) performed a survey based research and suggest solutions on natural convection in enclosures. Corcione (2003) numerically analyzed the natural convection heat transfer in a bottom surface heated rectangular enclosure for Rayleigh number interval of 10^3-10^6. Caronna et.al. (2009) numerically investigated bottom surface heated, upper surface cooled rectangular enclosure having cold and hot temperature sections on the opposite side walls. Natural convection in square enclosure having the half of side walls insulated and the other half being active is studied by Valencia and Frederick (1989) for Rayleigh number range of 10^3-10^7.

2. PROBLEM DEFINITION AND ANALYSIS

The conventional problem of natural convection in square cavity is stated as insulated horizontal walls and prescribed hot and cold surfaces at the opposite vertical walls. The problem of interest for such problem is to investigate Rayleigh number effect on heat transfer from the hot surface to the cold surface. For the present work, one of the vertical walls is kept high temperature and the top horizontal wall is insulated. Each of the other walls divided in two sections; one section of each wall insulated and other is kept low temperature. The arranged configurations are shown in Fig. 1. The uniform temperature boundary conditions are stated as one of the vertical wall is kept hot and the others are prescribed accordingly.



	Walls			
Configurations	1	2	3	4
1(Conventional)	C	C	A	A
2	C	A	C	A
3	C	A	A	C
4	A	C	C	A
5	A	C	A	C
6	A	A	C	C

Figure 1. Schematic drawing of the enclosure for CACA and the configurations to study

Steady state laminar natural convection flow is considered. The Boussinesq approximated non-dimensional form of the governing equations is given as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Ra\Pr\theta+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

Where the non-dimensional variables are;

$$X=\frac{x}{H},\ Y=\frac{y}{H},\ U=\frac{uH}{\alpha},\ V=\frac{vH}{\alpha}\ ,\ \theta=\frac{T-T_c}{T_h-T_c},\ P=\frac{pH^2}{\rho\alpha^2} \tag{5}$$

And the non-dimensional parameters are

$$Ra=\frac{g\beta(T_h-T_c)H^3}{\nu\alpha},\quad \Pr=\frac{\nu}{\alpha} \tag{6}$$

The boundary conditions;

$$U(X,0)=U(X,1)=U(0,Y)=U(1,Y)=0$$
$$V(X,0)=V(X,1)=V(0,Y)=V(1,Y)=0$$
$$\theta(0,Y)=1 \qquad \frac{\partial\theta}{\partial Y}(X,1)=0$$

Thermal boundary conditions for the prescribed boundary cases;

Case 1: $\theta(1,Y)=0$ $\qquad \frac{\partial\theta}{\partial Y}(X,0)=0$

Case 2: $\theta(1,Y)=0$ 0.5<Y<1 $\quad \frac{\partial\theta}{\partial X}(1,Y)=0$ 0<Y<0.5

$\theta(X,0)=0$ 0.5<X<1 $\quad \frac{\partial\theta}{\partial Y}(X,0)=0$ 0<X<0.5

Case 3: $\theta(1,Y)=0$ 0.5<Y<1 $\quad \frac{\partial\theta}{\partial X}(1,Y)=0$ 0<Y<0.5

$\theta(X,0)=0$ 0<X<0.5 $\quad \frac{\partial\theta}{\partial Y}(X,0)=0$ 0.5<X<1

Case 4: $\theta(1,Y)=0$ 0<Y<0.5 $\quad \frac{\partial\theta}{\partial X}(1,Y)=0$ 0.5<Y<1

$\theta(X,0)=0$ 0.5<X<1 $\quad \frac{\partial\theta}{\partial Y}(X,0)=0$ 0<X<0.5

Case 5: $\theta(1,Y)=0$ 0<Y<0.5 $\quad \frac{\partial\theta}{\partial X}(1,Y)=0$ 0.5<Y<1

$\theta(X,0)=0$ 0<X<0.5 $\quad \frac{\partial\theta}{\partial Y}(X,0)=0$ 0.5<X<1

Case 6: $\theta(X,0)=0$ $\qquad \frac{\partial\theta}{\partial X}(1,Y)=0$

The well known stream function and Bejan's heat function are defined as

$$U=\frac{\partial\psi}{\partial Y},\quad V=-\frac{\partial\psi}{\partial X} \tag{7}$$

and

$$\left.\begin{aligned} -\frac{\partial\Pi}{\partial X}&=V\theta-\frac{\partial\theta}{\partial Y}\\ \frac{\partial\Pi}{\partial Y}&=U\theta-\frac{\partial\theta}{\partial X}\end{aligned}\right\} \tag{8}$$

respectively.

The set of equations (1)-(4) were discretized by finite control volume method. A FORTRAN based elliptic program is modified to solve the discretized equation via the SIMPLE algorithm by Patankar (1980). Grid refinement study was performed for 20x20, 40x40, 80x80, 120x120, 160x160, 240x240 control volumes. It is observed that 120x120 grid sizing result well matches with Davis's (1983) benchmark solution as compared in Fig 2.

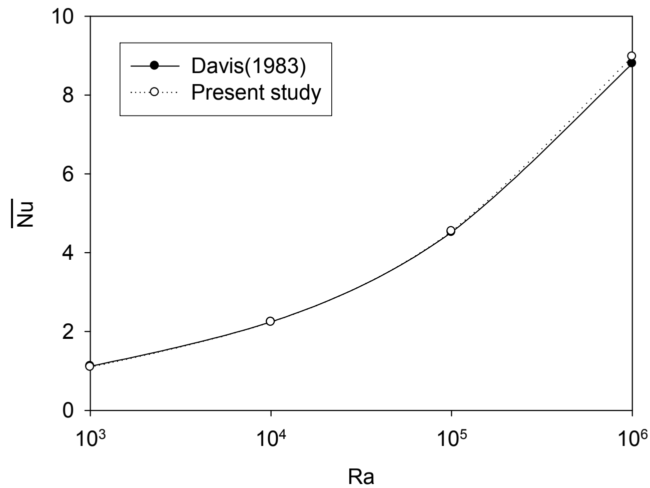


Figure 2. Present study and Davis's (1983) benchmark result for 120x120 control volumes

The local and mean Nusselt numbers are defined as

$$Nu_l=-\left.\frac{\partial\theta}{\partial X}\right|_{X=0}\qquad \overline{Nu_l}=\frac{1}{H}\int_0^1 Nu_l dY$$

3. RESULTS AND DISCUSSION

The numerical results are obtained for Rayleigh number interval of 10^4 - 10^6, and Prandtl number value of 0.71. For the case 1, referred as the conventional natural convection, streamlines, isotherms and the heatlines are presented for three different Rayleigh numbers in Fig. 3. Single flow cell is doubled with increasing Rayleigh number and are positioned upper of the left wall and lower of the right wall. Corresponding isotherms are becoming fine with increasing Ra number near to the vertical surfaces. Following to the streamlines and the isotherms, the heatlines presents the expected result; the lines become fine with increasing Rayleigh number which indicates increment in heat flux. This result is also agreed with the numerical results obtained by Davis (1983).

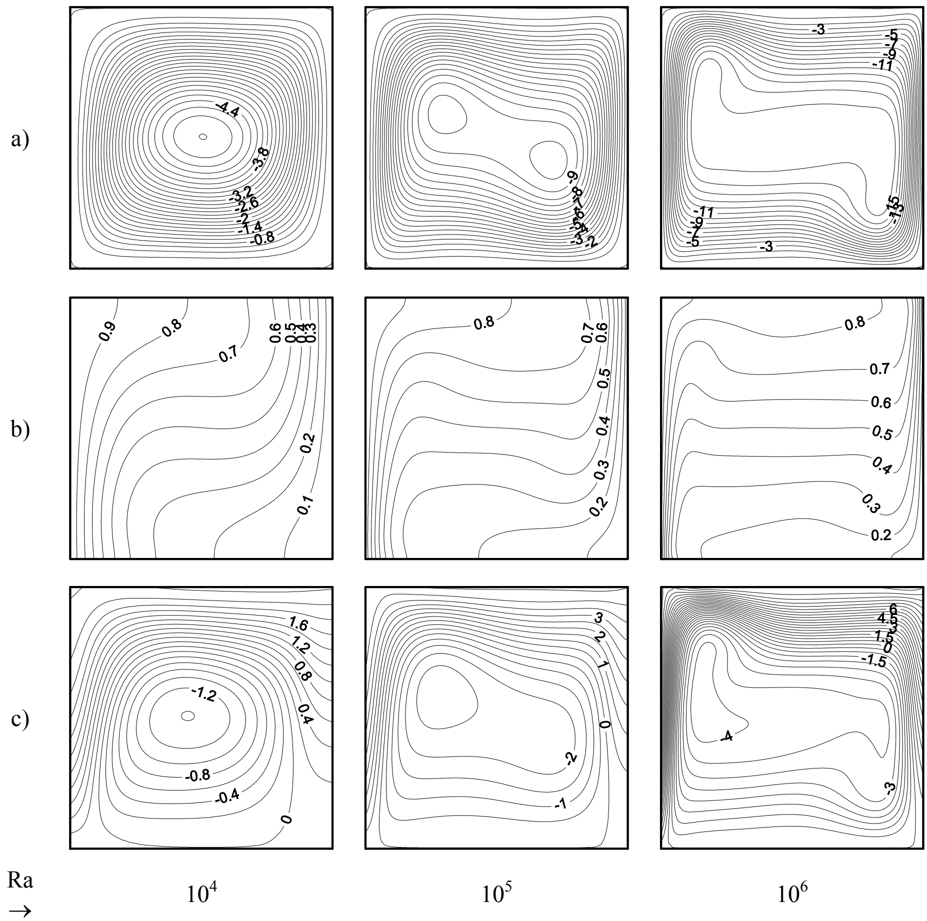


Figure 3. Stream line (a), isotherm (b) and heatline (c) maps for case 1

The obtained streamline patterns for six different wall configurations and three different Rayleigh numbers are given in Fig. 4. All configurations except configuration AACC tend to perform double recirculation zones as Rayleigh numbers increases.

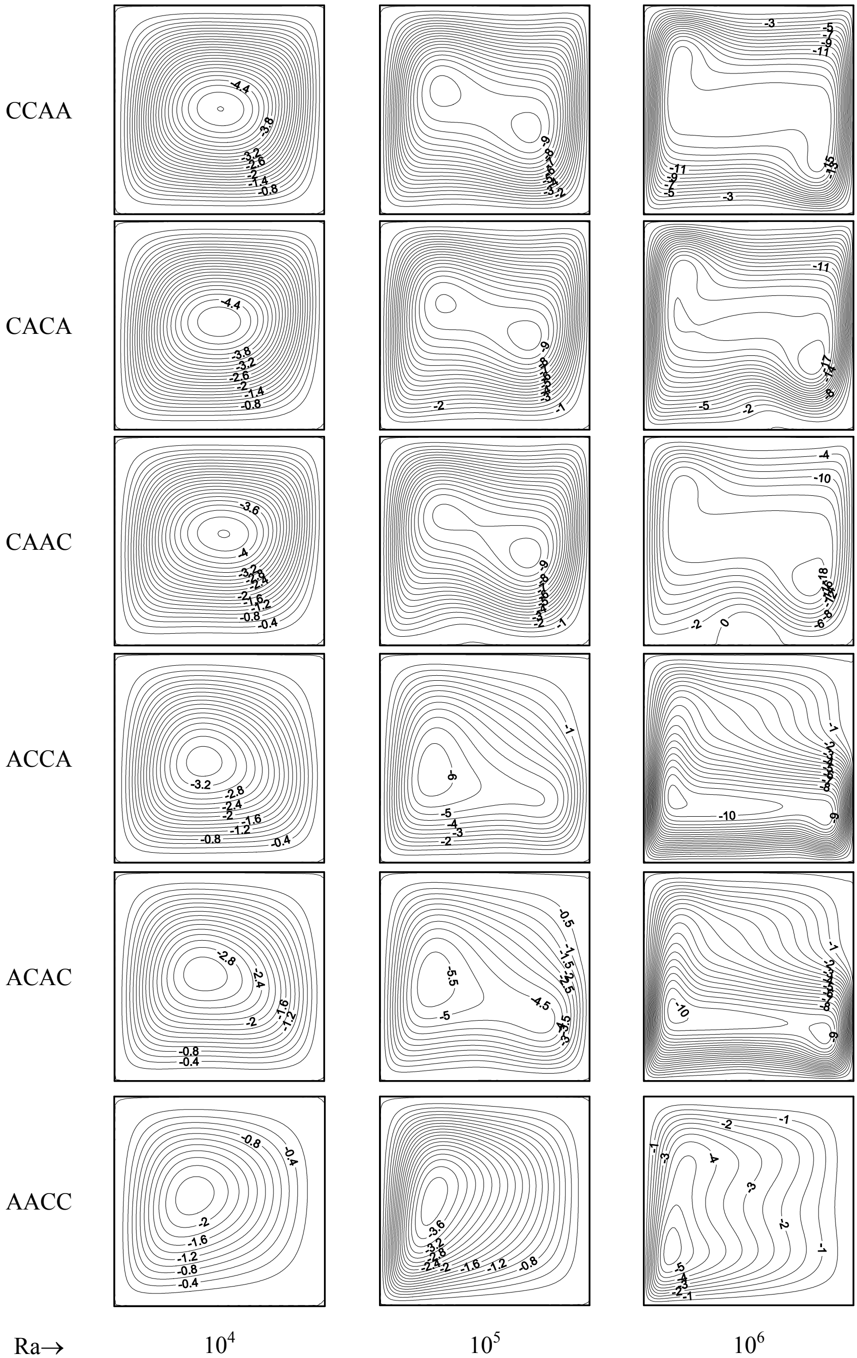


Figure 4. The streamline patterns for different wall configurations and Rayleigh numbers

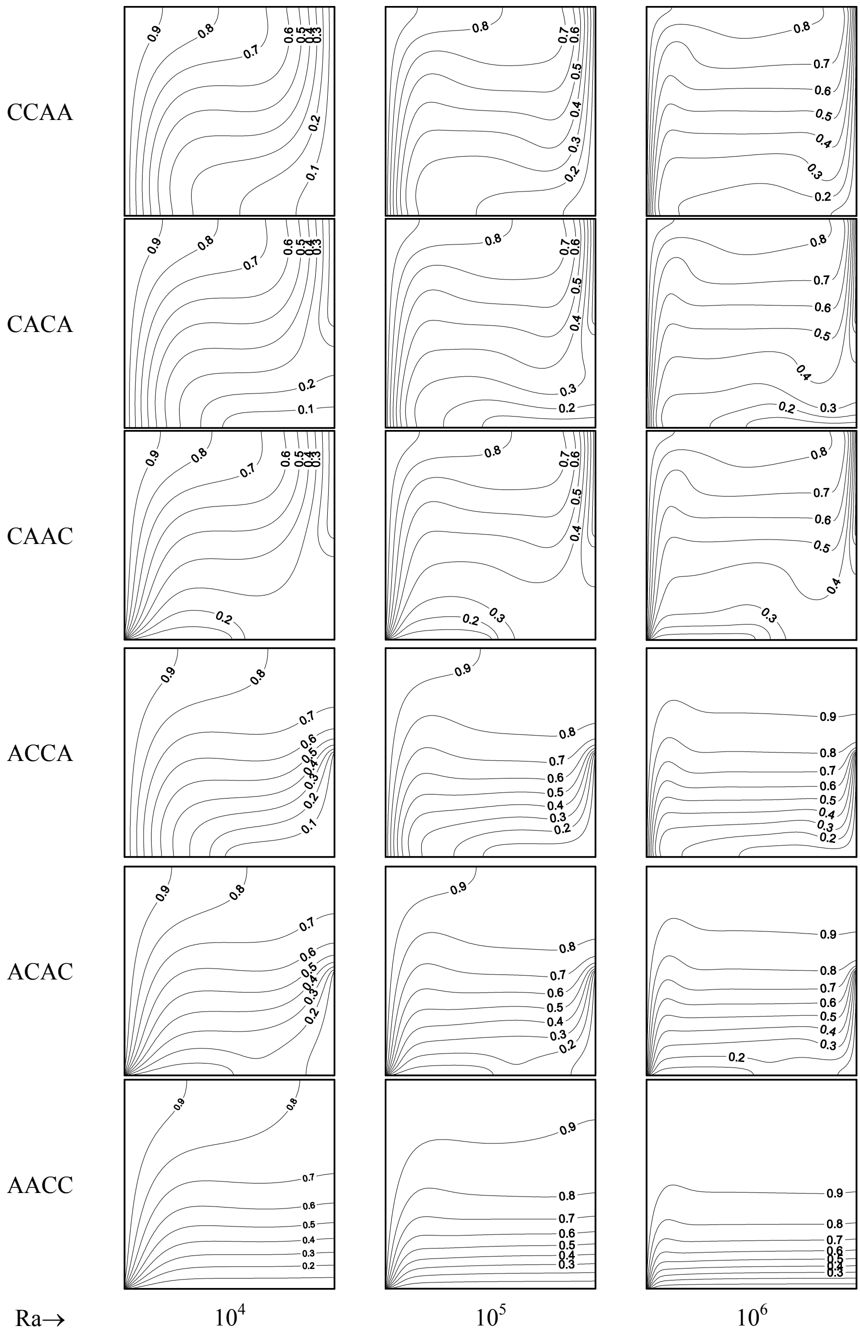


Figure 5. The isotherm patterns for different wall configurations and Rayleigh numbers

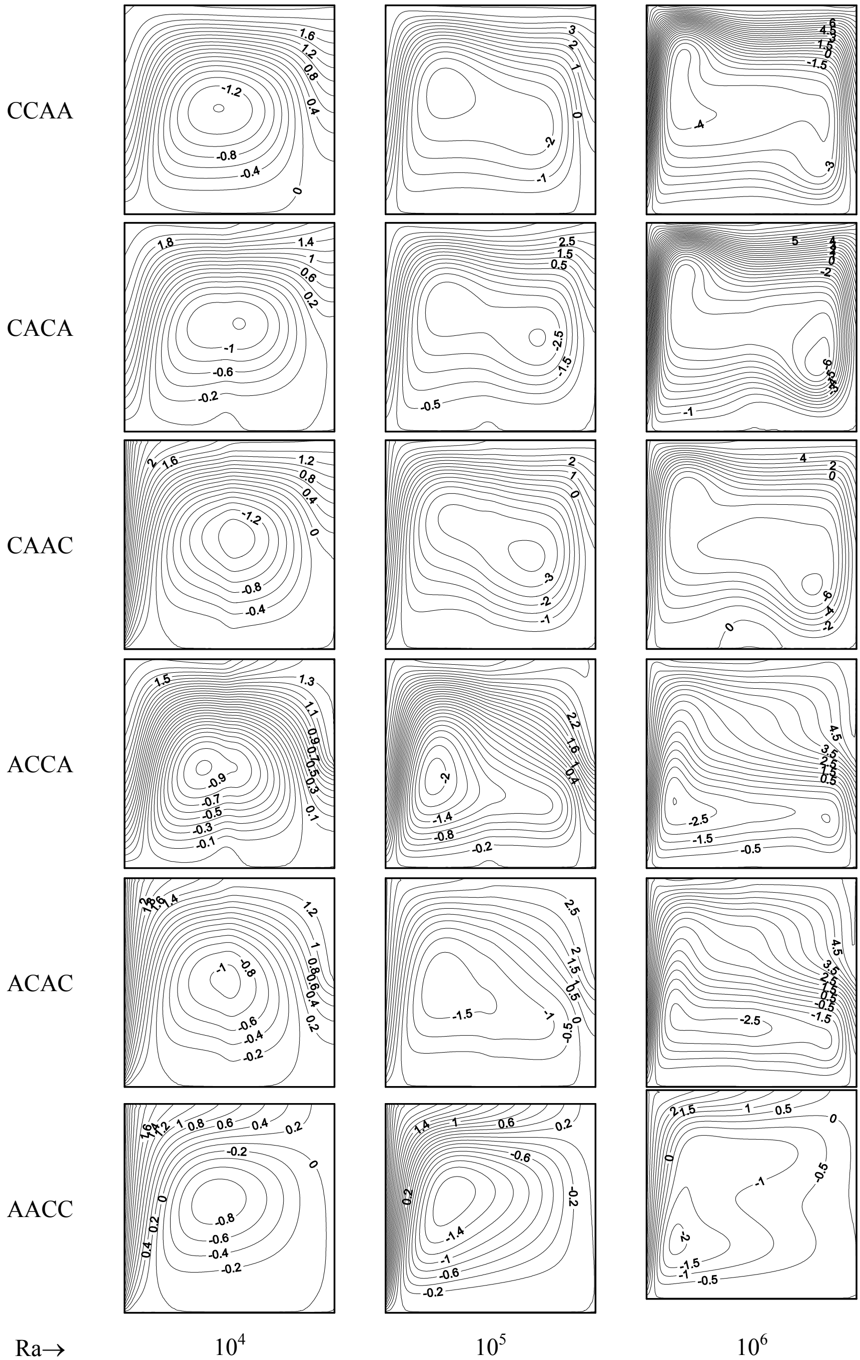


Figure 6. The heatline patterns for different wall configurations and Rayleigh numbers

The corresponding isotherms are shown in Fig. 5. For the last three configurations (ACCA, ACAC, AACC), the enclosure is almost filled with the hot fluid, whereas for the first three cases, the upper region of the enclosure is dominated by cold fluid.

Heatline patterns for the configurations are given in Fig. 6. Heatlines grow around the center of the enclosure for low values of Rayleigh numbers. Heatlines tend to move vertical wall region with increasing Rayleigh number.

The mean Nusselt number variation with Rayleigh number at left vertical wall for several configurations is shown in Fig. 7. It is shown that the highest mean Nusselt number value is observed for configuration CAAC, whereas the lowest value is observed for configuration ACCA. It is also observed that the configuration AACC is less sensitive to Rayleigh number. On the other hand, the configuration CCAA shows the strongest dependence on Rayleigh number. The other configurations remain between two extreme cases.

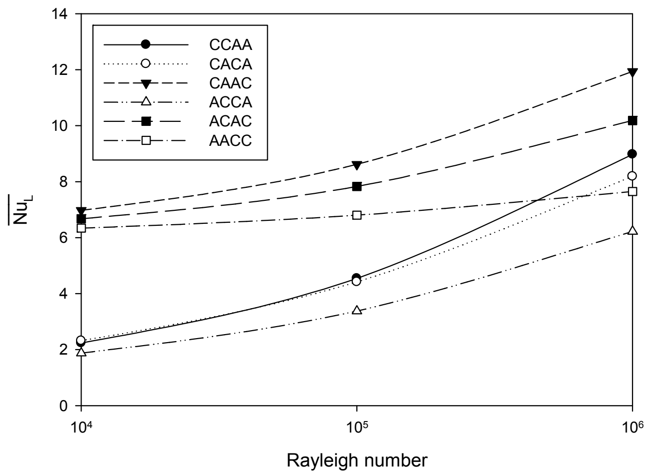


Figure 7. The mean Nusselt number at left wall versus Rayleigh number for the prescribed configurations

Variation of local Nusselt number for Ra = 10^4 along the left vertical wall is shown in Fig. 8. Nusselt number variation for configurations ACCA, CCAA and CACA are different than the variations for the configurations AACC, ACAC and CAAC. For all the cases local Nusselt number tends to decrease from the lower wall to upper wall. Approximately after vertical location Y = 0.4, the Nusselt number curves show almost similar behavior.

Local Nusselt number variation for Ra=10^5 is shown in Fig. 9. Starting from the vertical location Y=0.3, Nusselt numbers for all configurations show the similar trend and varies in decreasing manner along the vertical direction. Nusselt number variation for configuration AACC performs significant change between bottom and top along the left wall.

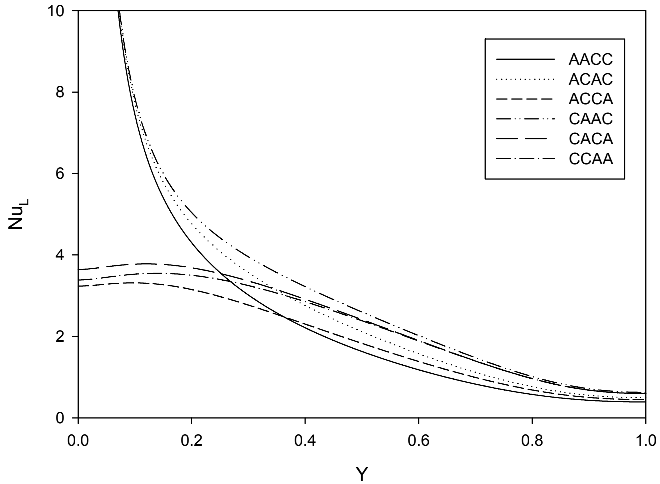


Figure 8. Variation of local Nusselt number along the left vertical wall for Ra=10^4

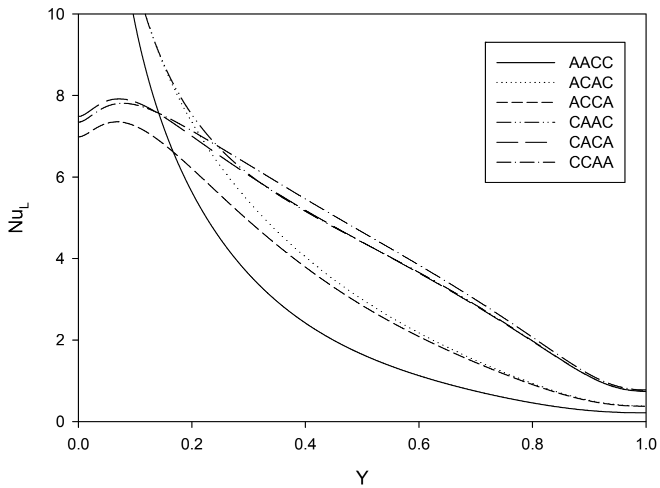


Figure 9. Variation of local Nusselt number along the left vertical wall for Ra=10^5

Local Nusselt number variation across the vertical wall for Ra = 10^6 is shown in Fig. 10. Trends are very similar to case of Ra=10^5, but the local Nusselt number values are higher than that of Ra=10^5 as expected.

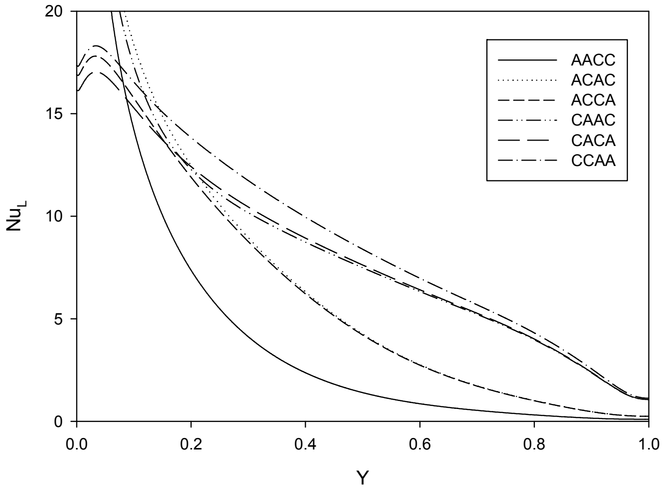


Figure 10. Variation of local Nusselt number along the left vertical wall for $Ra=10^6$

4. CONCLUSIONS

Natural convection heat transfer problem in a square enclosure is considered under the prescribed boundary conditions; the upper wall is insulated and one of the side walls is kept uniform high temperature while the other two walls are configured as mixed of partially insulated/uniform cold temperature. Six different configuration of the boundary condition has been numerically investigated. From the computational results of this investigation, the following conclusions can be drawn.

(1) The highest mean Nusselt number values for the left wall occurs for case CAAC; as upper half of the vertical wall and left half of the bottom wall are kept uniform cold temperature. Comparing with the conventional natural convection (case CCAA), 33 % of increment in the mean Nusselt number is observed.

(2) The lowest mean Nusselt number values for the left wall occurs for case ACCA; as lower half of the vertical wall and right half of the bottom wall are kept uniform cold temperature. Comparing with the conventional natural convection (case CCAA), 30 % of decrement in the mean Nusselt number is observed.

(3) When all of the bottom wall cold and insulated right wall are considered (case AACC), Rayleigh number becomes almost ineffective on the mean Nusselt number.

(4) Cases CAAC and ACAC perform higher heat transfer comparing with case AACC within the prescribed Rayleigh number interval. Result of the heat transfer for cases ACCA and CACA remain below, conventional natural convection, the case CCAA,.

(5) Increasing Rayleigh number generates a second recirculation zone except case AACC which is less sensitive to the Rayleigh number.

(6) As a final concluding remark, a proper prescription of bottom and side wall boundary conditions has capability of satisfying the user intended target in terms of heat transfer performance.

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