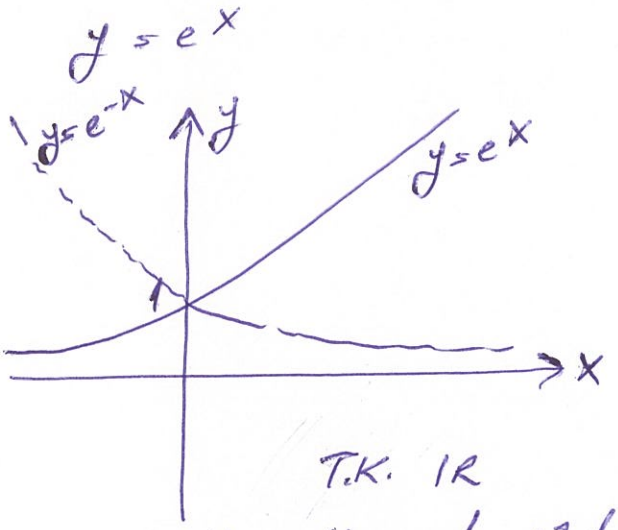
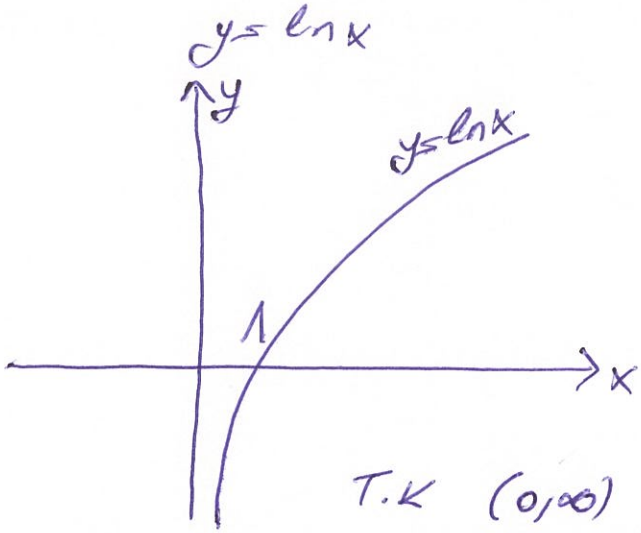
Üstel ve Logaritmik Fonksiyonların Limitleri

$e \approx 2{,}718\ldots$



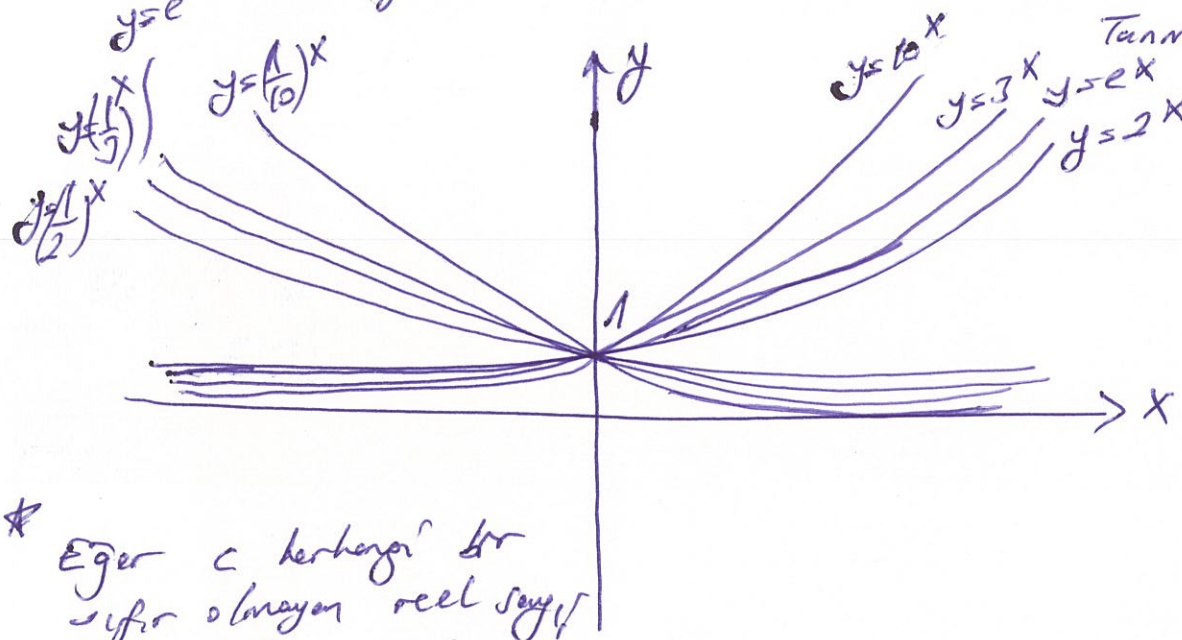
T.K. $\mathbb{R}$

Heryerde sürekli



T.K $(0,\infty)$

Tanım kümesinde sürekli



★ Eğer c herhangi bir sıfır olmayan reel sayı ise

$$\lim_{x \to c} e^x = e^c$$

$$\lim_{x \to c} 2^x = 2^c$$

$$\lim_{x \to +\infty} e^x = +\infty$$

$$\lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to +\infty} e^{-x} = \lim_{x \to \infty} \frac{1}{e^x} = \frac{\lim_{x \to \infty} 1}{\lim_{x \to \infty} e^x} = \frac{1}{\infty} = 0$$

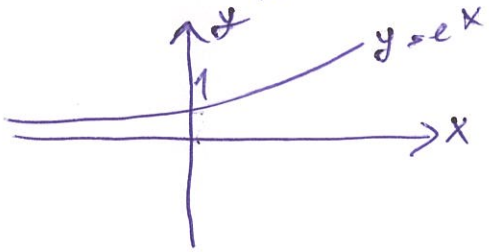
$$\lim_{x \to -\infty} e^{-x} = \lim_{x \to -\infty} \frac{1}{e^x} \quad \begin{matrix} -x = t \\ x = -\infty \end{matrix} \quad \begin{matrix} t = +\infty \Rightarrow \\ \lim_{t \to \infty} e^t = +\infty \end{matrix}$$

ÖRNEK

$\lim_{x\to 0^+} e^{1/x} \Rightarrow \lim_{t\to\infty} e^t = +\infty$

$\frac{1}{x} = t$ olsun.

$x \to 0^+ \Rightarrow \frac{1}{x} \to +\infty$

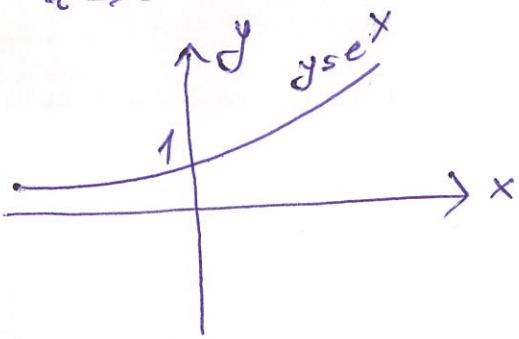


$\lim_{x\to\infty} e^{\frac{1}{x}} \Rightarrow 1$ ✓

$\frac{1}{x} = t$

$x \to \infty \Rightarrow t = 0$

$\lim_{t\to 0} e^t = e^0 = 1$

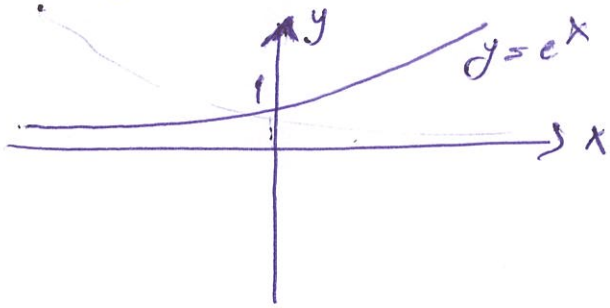


$\lim_{x\to 0^-} e^{\frac{1}{x}} =$

$x \to 0^- \Rightarrow t = \frac{1}{x}$

$t \to -\infty$

$\lim_{t\to -\infty} e^t = 0$



$\lim_{x\to\infty} e^{1/x}$

Pratik $e^{-\frac{1}{\infty}} = e^{-0} = e^0 = 1$

$\frac{1}{x} = t \Rightarrow x \to -\infty \Rightarrow t = 0$

$\lim_{t\to 0} e^t \Rightarrow \lim_{t\to 0} e^0 = 1$ ✓

Eğer c herhangi bir negatif veya sıfır olmayan reel sayı ise;

$\lim_{x\to c} \log x = \log c$

$\lim_{x\to c} \ln x = \ln c$

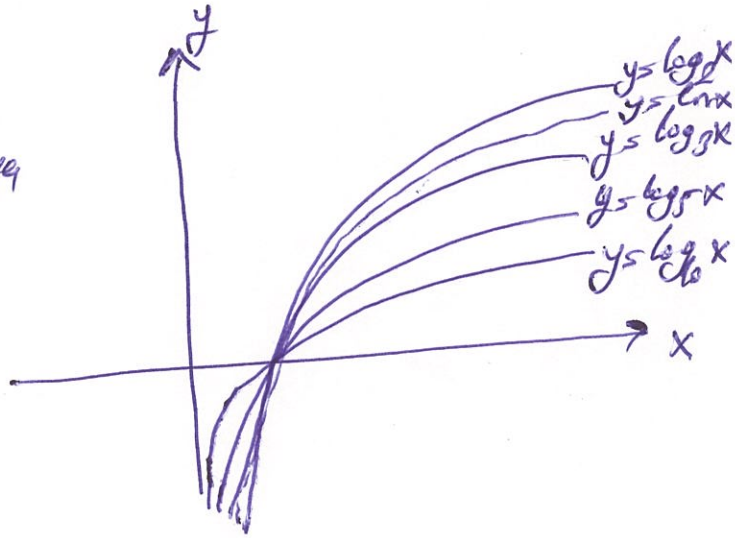
$\lim_{x\to\infty} \ln x = +\infty$

$\lim_{x\to -\infty} \ln x =$ Tanımsız

$\lim_{x\to 0^+} \ln x = -\infty$

$\lim_{x\to 0^-} \ln x =$ Tanımsız

$\lim_{x\to 0} \ln x =$ YOK

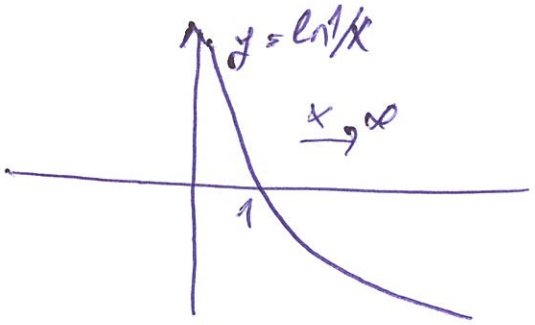


$$\lim_{x\to\infty} \ln \frac{1}{x}$$

$$\lim_{x\to\infty} \ln x^{-1}$$

$$\lim_{x\to\infty} (-\ln x) = -\underbrace{\lim_{x\to\infty} \ln x}_{\infty}$$

$$= -\infty$$



$y = \ln 1/x \to -\infty$

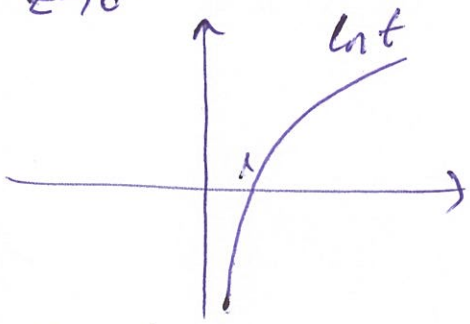
$$\lim_{x\to\infty} \ln \frac{2}{x^2}$$

$$\frac{2}{x^2} = t$$

$$x^2 < \infty \Rightarrow \underbrace{\frac{2}{x^2}}_{t} > \frac{2}{\infty} = 0$$

$$t > 0$$

$$\lim_{t\to 0^+} \ln t = -\infty$$



2. yol

$$\lim_{x\to\infty} \ln\left(\frac{2}{x^2}\right) = \lim_{x\to\infty} (\ln 2 - \ln x^2)$$

$$\lim_{x\to\infty} (\ln 2 - 2\ln x)$$

$$\lim_{x\to\infty} \ln 2 - 2 \underbrace{\lim_{x\to\infty} \ln x}_{\infty}$$

$$= \ln 2 - 2\cdot\infty = -\infty$$

$$\lim_{x\to -\infty} \ln \frac{1}{x} = \text{Tanımsız}$$

negatif

$$\lim_{x\to 0^+} \ln \frac{2}{x^2} = \lim_{x\to 0^+} (\ln 2 - 2\ln x)$$

$$= \lim_{x\to 0^+} \ln 2 - 2 \lim_{x\to 0^+} \ln x$$

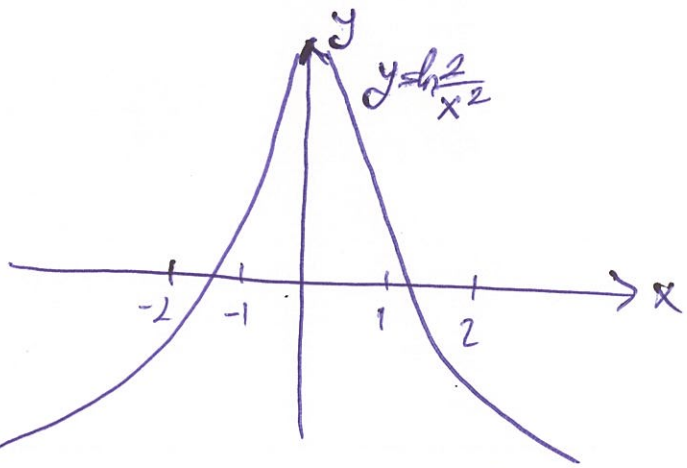
$$= \ln 2 - 2(-\infty) = \ln 2 + \infty = \infty$$

2. yol

$$x \to 0^+ \Rightarrow \frac{1}{x^2} = +\infty$$

$$\frac{2}{x^2} = 2\cdot +\infty = \infty$$

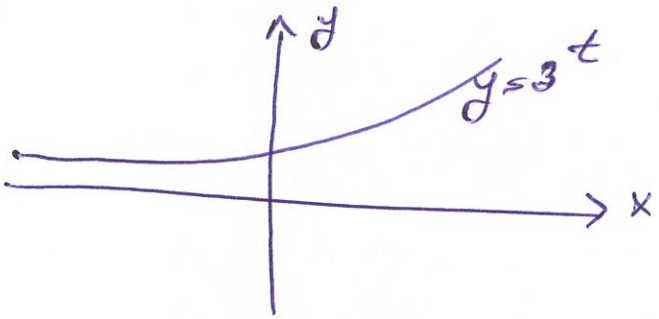
$$\lim_{t\to\infty} \ln t = \infty$$



$$\lim_{x\to 0^+} \frac{1}{1+3^{1/x}} = \frac{\lim_{x\to 0^+} 1}{\lim_{x\to 0^+} 1 + \lim_{x\to 0^+} 3^{\frac{1}{x}}} = \frac{1}{1+\infty} = 0$$

$$\lim_{x\to 0^+} 3^{1/x} = \lim_{t\to\infty} 3^t = +\infty$$

$x \to 0^+ \Rightarrow \frac{1}{x} \to \infty$



$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\lim_{x\to\infty} \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \quad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} \frac{\frac{e^{2x}-1}{e^x}}{\frac{e^{2x}+1}{e^x}} = \lim_{x\to\infty} \frac{e^{2x}-1}{e^x} \cdot \frac{e^x}{e^{2x}+1} = \lim_{x\to\infty} \frac{e^{2x}-1}{e^{2x}+1} \quad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} \frac{e^{2x}\left(1-\frac{1}{e^{2x}}\right)}{e^{2x}\left(1+\frac{1}{e^{2x}}\right)} = 1$$

$$\lim_{x\to\infty} \ln\left(\frac{x}{x+8}\right) = \ln\left(\underbrace{\lim_{x\to\infty} \frac{x}{x+8}}_{1}\right) = 0$$

$$\lim_{x\to\infty} \frac{x}{x+8} = \lim_{x\to\infty} \frac{x}{x\left(1+\frac{8}{x}\right)} = 1 \checkmark$$

$$\lim_{x\to\infty} \left(\frac{\ln 2x}{\ln 3x}\right) = \left(\frac{\infty}{\infty}\right) \underset{t\to\ln x}{=} \lim_{t\to\infty} \frac{\ln 2 + t}{\ln 3 + t} = \lim_{t\to\infty} \frac{t\left(\frac{\ln 2}{t}+1\right)}{t\left(\frac{\ln 3}{t}+1\right)}$$

$x \to \infty \Rightarrow t \to \infty$

$$\frac{\ln 2x}{\ln 3x} = \frac{\ln 2 + \ln x}{\ln 3 + \ln x} = 1 \checkmark$$

KAYNAKLAR

1- MathProf, Youtube Kanalı, 2025.