Trigonometrik Fonksiyonların Türevi

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

ÖRNEK

$y = x \sin x$ ise $\frac{dy}{dx} = ?$

$$y' = x' \cdot \sin x + x \cdot (\sin x)'$$

$$y' = \sin x + (x \cdot \cos x)$$

$$\frac{d}{dx}(\tan x) = \left(\frac{\sin x}{\cos x}\right)'$$

$$= \frac{(\sin x)' \cdot \cos x - (\sin x \cdot \cos x')}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{(\cos x)^2} \qquad \frac{1}{\cos x} = \sec x$$

$$= \sec^2 x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x)$$

$$= \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{(-\sin x) \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} \qquad \frac{1}{\sin x} = \operatorname{cosec} x$$

$$= -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{[0] \cdot \cos x - [-\sin x] \cdot 1}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \cdot \sec x$$

$$\frac{d}{dx}\left(\frac{1}{\sin x} = \operatorname{cosec} x\right) = \frac{[0] \cdot \sin x - [\cos x] \cdot 1}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \operatorname{cosec} x$$

ÖRNEK

$y = (4\sqrt{x} - 3\sqrt[3]{x})\cos x$ ise $y' = ?$

$y' = [4\sqrt{x} - 3\sqrt[3]{x}]' \cos x + [\cos x]' \cdot (4\sqrt{x} - 3\sqrt[3]{x})$

$y' = \left(4 \cdot \frac{1}{2\sqrt{x}} - 3 \cdot \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}\right)\cos x + (-\sin x) \cdot (4\sqrt{x} - 3\sqrt[3]{x})$

$y' = \left(\frac{2}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}}\right)\cos x + (-\sin x) \cdot (4\sqrt{x} - 3\sqrt[3]{x})$

ÖRNEK

$y = (x^4)\sin x \cdot \tan x$ ise $y' = ?$

$y' = [(x^4)\sin x]' \tan x + [\tan x]' (x^4)\sin x$

$= [[4x^3]\sin x + [\cos x]\, x^4] \tan x + [\sec^2 x]\, x^4 \sin x$

ÖRNEK

$y = \frac{\sec x}{1+\tan x}$ ise $y' = ?$

$y' = \frac{[\sec x]'(1+\tan x) - [1+\tan x]' \sec x}{(1+\tan x)^2}$

$= \frac{[\sec x \tan x](1+\tan x) - [\sec^2 x]\sec x}{(1+\tan x)^2}$

$=$

$\frac{d}{dx}\left(\frac{\sin x - x\cos x}{\cos x + x\sin x}\right)$

$= \frac{(\sin x - x\cos x)' \cdot [\cos x + x\sin x] - (\sin x - x\cos x) \cdot (\cos x + x\sin x)'}{(\cos x + x\sin x)^2}$

$= \frac{\cos x - (1\cdot\cos x) + x\sin x\,[\cos x + x\sin x] - (\sin x - x\cos x) \cdot (-\sin x + x\cos x)}{(\cos x + x\sin x)^2}$

$= \frac{x\cdot\sin x\,(\cos x + 1) - (\sin x - x\cos x)(-\sin x + x\cos x)}{(\cos x + x\sin x)^2}$

$$\frac{d}{dw}\left(\cos w \log_3 w\right) = ?$$

$$= (\cos w)' \cdot \log_3 w + \cos w \cdot (\log_3 w)'$$

$$= -\sin x \log_3 w + \cos w \cdot \frac{1}{\ln 3}\frac{1}{w}$$

$$= -\sin x \log_3 w + \frac{1}{w \ln 3}\cos w$$

Eğer $f(x) = \ln x^2 + 4 \ln x \cos x$ ise $f'(x) = ?$

$$f'(x) = (2\ln x)' + (4 \ln x \cos x)'$$

$$= \frac{2}{x} + 4 \cdot \left[\frac{1}{x}\cos x + \ln x \cdot (-\sin x)\right]$$

$$= \boxed{\frac{2}{x} + \frac{4}{x}\cos x - 4 \ln x \sin x}$$

$$\frac{d}{dx}\left(\frac{1-\sin x}{1+\cos x}\right)$$

$$= \frac{(1-\sin x)' \cdot (1+\cos x) - (1-\sin x) \cdot (1+\cos x)'}{1+\cos x}$$

$$= \frac{(-\cos x)(1+\cos x) - (1-\sin x)(-\sin x)}{1+\cos x}$$

$$= \frac{-\cos x - \cos^2 x + \sin x - \sin^2 x}{1+\cos x}$$

$$= \frac{-\cos x + \sin x - 1}{1+\cos x}$$

$$= \frac{-(-\sin x + 1 + \cos x)}{1+\cos x}$$

$$= \boxed{\frac{\sin x}{1+\cos x} - 1}$$

ÖRNEK

$$g(x) = \tan(x^2+1)\cdot\sec(\ln(x))$$

$$= \tan(x^2+1)'\sec(\ln(x)) + \tan(x^2+1)\cdot\sec(\ln x)'$$

$$= \sec(x^2+1)\cdot(2x)\sec(\ln(x)) + \tan(x^2+1)\cdot\sec(\ln x)\tan(\ln$$

$$= 2x\sec^2(x^2+1)\sec(\ln(x)) + \frac{\tan(x^2+1)\sec(\ln x)\tan(\ln x)\cdot\frac{1}{x}}{x}$$

ÖRNEK

$$f(x) = \frac{\sin(x)\cdot\tan(x)}{\cos(x^2)}$$

$$f'(x) = \frac{(\sin x\tan x)'\cos(x^2) - \sin x\cdot\tan x\cdot(\cos(x^2)')}{\cos(x^2)^2}$$

$$= \frac{(\cos x\cdot\tan x + \sin x\cdot\sec^2 x)\cos(x^2) + \sin x\cdot\tan x\cdot 2x\cdot\sin(x^2)}{(\cos(x^2))^2}$$

$$= \frac{\cos(x)\tan x\cos(x^2) + \sin x\sec^2(x)\cos(x^2) + 2x\sin(x)\tan x\sin($$

Zincir Kuralı

İç içe geçmiş fonksiyonlara bileşke fonksiyon denir.

$$(f\circ g\circ t\circ m)(x) = f(g(t(m(x))))$$

$\sin(\sqrt{\ln(x^2)})$

NOT: Bileşke bir fonksiyonun türevini alabilmek için dıştaki fonksiyonun önce türevi alınır ve iç kısımdakiler aynen korunur sonra içteki fonksiyonlarla aynı işlem uygulanır.

Genel Kuvvet Kuralı

Eğer $u(x)$ türevlenebilir bir fonksiyon ise, n herhangi bir reel sayı ve $y = f(x) = (u(x))^n$ ise;

$$f'(x) = [n \cdot (u(x))^{n-1}][u'(x)]$$

ÖRNEK

$f(x) = (\sin x + 4)^7$ ise $f'(x) = ?$

$= 7 \cdot (\sin x + 4)^6 \cdot (\sin x + 4)'$

$= 7 \cdot (\sin x + 4)^6 \cdot \cos x$

$\frac{d}{dr}\left(\frac{1}{r^2+2r+3}\right)^3$

$= \left((r^2+2r+3)^{-3}\right)'$

$= -3 \cdot (r^2+2r+3)^{-4} \cdot \underbrace{(r^2+2r+3)'}_{(2r+2)}$

$h(w) = \sqrt{6-w}$ ise $h'(w) = ?$

$((6-w)^{1/2})'$

$\frac{1}{2}(6-w)^{-1/2} \cdot \underbrace{(6-w)'}_{(-1)}$

$= -\frac{1}{2}\frac{1}{\sqrt{6-w}}$

$(\sin^5 x)' = \left((\sin x)^5\right)' = 5 \cdot (\sin x)^4 \cdot \underbrace{(\sin x)'}_{\cos x}$

$(\sin x^6)' = (\cos x^6) \cdot \underbrace{(x^6)'}_{6x^5}$

$(\ln(x^3+1))' = \frac{1}{x^3+1} \cdot \underbrace{[x^3+1]'}_{3x^2}$

$e^{(x^2 \sin x)} = e^{x^2 \sin x} \cdot (x^2 \sin x)'$

$= e^{x^2 \sin x} \cdot (2x \sin x + x^2 \cos x)$

Kaynaklar

1- MathProf, 2025.

2- Genel Matematik, Mustafa BALCI, 2018.