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Physics-informed neural network and data-driven modeling of non-Fourier heat transfer in laser-irradiated semiconductor media using bi-Helmholtz nonlocal theory

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Abstract This study develops a hybrid analytical and physics-informed machine learning framework to analyze non-Fourier heat transfer in laser-irradiated semiconductor media. The model integrates bi-Helmholtz nonlocal thermoelasticity with dual length-scale parameters to capture size-dependent mechanical effects, alongside a modified Green–Naghdi heat conduction theory to describe finite-speed thermal wave propagation and relaxation phenomena. The coupled governing equations for displacement, temperature, carrier density, and stress are first solved analytically using a normal-mode approach to obtain benchmark solutions. A physics-informed neural network (PINN) is then constructed by embedding the governing equations and boundary conditions into the learning process, enabling efficient and accurate prediction of multiphysics responses. The proposed approach significantly reduces computational cost while preserving high accuracy. Parametric analysis highlights the strong influence of nonlocal parameters and thermal relaxation on wave propagation and carrier dynamics. The framework offers a robust tool for real-time simulation of laser-induced thermal processes in semiconductor systems.

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1 Introduction

Semiconductors form the backbone of modern technology due to their tunable optical, electrical, and thermal properties, enabling diverse electronic, photonic, and sensing applications. Recent studies have increasingly focused on fractional-order and thermoelastic models, which more accurately capture the complex multiphysical behavior of semiconductor materials. Photothermoelastic analysis has gained significant importance because of its applications in photoacoustic microscopy, thermal imaging, and nondestructive evaluation. Under laser irradiation or transient thermal excitation, photoexcited carriers generate localized heating and mechanical deformation, leading to coupled thermoelastic and plasma wave propagation within the semiconductor medium [1–3]. A semiconductor characterized by an energy band gap E_g is subjected to laser irradiation with photon energy E , such that $E > E_g$. In this process, absorbed photons excite electrons, enabling their transition from the valence band to the conduction band. This electronic transition occurs only when the photon energy meets or exceeds the band gap requirement, i.e., $E \geq E_g$ [4–7].

Traditional continuum theories, such as linear elasticity, inherently neglect size-dependent effects. Consequently, these models predict non-dispersive wave propagation and remain valid only for small wave numbers. Moreover, in the presence of material defects such as dislocations and disclinations, classical elasticity leads to singular behavior in the elastic strain, stress, and strain energy near the defect cores [8,9]. Such singularities are physically unrealistic and highlight the limitations of classical elasticity in accurately describing material behavior at microscopic scales [10,11]. A series of investigations has contributed to the evolution of nonlocal continuum field theory [12–17]. Eringen [18] significantly advanced this framework and contributed extensively to its development [19–29]. This theory is based on the concept that the behavior at any material point is influenced by the state of all other points within the body [18,19,27,30]. In nonlocal elasticity, the stress at a point is described through an integral constitutive relation involving the convolution of an attenuation function with the macroscopic stress field, providing a physically meaningful representation [29]. However, this integral form leads to a first-kind Fredholm equation, which is generally difficult to solve due to its ill-posed nature. Ill-posedness commonly arises when weakly singular or continuous kernels are adopted to describe attenuation functions [31]. Eringen [29] demonstrated that, for a specific class of attenuation functions in an infinite domain, the integral formulation can be transformed into an equivalent differential form. This differential representation can be expressed compactly using a Helmholtz-type operator, to which a clear physical interpretation was assigned [18,29]. Subsequently, the nonlocal differential beam model has been examined in several studies [32,33]. Investigations into its dynamic response [34–36] revealed that the model generally predicts a softening behavior compared to the classical local theory in benchmark problems. Nevertheless, studies revealed that the fundamental eigenfrequency of a cantilever beam is higher than the classical prediction, giving rise to an early paradox in the differential formulation [35].

The Modified Green–Naghdi (MGN) model is an advanced non-Fourier heat conduction theory developed to extend the classical Green–Naghdi thermoelastic framework for ultrafast and nanoscale heat transport. Unlike Fourier’s law, the GN theory introduces thermal displacement to describe finite-speed thermal wave propagation, thereby eliminating the paradox of infinite heat propagation. However, the classical GN formulations remain limited in capturing thermal relaxation, dispersive wave behavior, and memory-dependent effects under short-pulse and high-gradient thermal excitations. The MGN model overcomes these limitations by incorporating multiple relaxation times, higher-order heat flux derivatives, and hereditary memory effects, providing a more accurate description of transient heat transfer processes. Due to its analogy with viscoelastic constitutive models such as Burgers and Kelvin–Voigt models, the MGN framework offers a physically consistent approach for modeling thermal dissipation, dispersion, and relaxation in generalized thermoelastic media [37]. Moreover, the generalized formulation unifies wave-type and diffusion-type heat conduction and includes the Fourier, Cattaneo–Vernotte, Lord–Shulman, and standard Green–Naghdi theories as special limiting cases. Its hyperbolic governing equations ensure finite thermal wave speeds and enable accurate analysis of transient thermoelastic phenomena in ultrafast laser heating, low-temperature environments, and micro- and nanoscale heat transport.

The MGN theory is highly relevant in photothermal semiconductor analysis, particularly for laser-matter interactions and microscale heat transport. Unlike the classical Fourier law, the MGN framework incorporates finite thermal wave speeds, relaxation effects, and thermal memory, enabling a more accurate description

of transient thermoelastic behavior in semiconductors. It is especially useful in stochastic photothermoelastic modeling, where uncertainties in material properties, carrier generation, boundary conditions, and energy absorption significantly influence the response. The theory enables reliable prediction of temperature fields, photoinduced stresses, and carrier-induced deformations under fluctuating thermal conditions. Consequently, the MGN model finds important applications in semiconductor devices, optoelectronic systems, laser diagnostics, and advanced materials processing.

Recent studies have extensively examined photothermal and semiconductor media. In contrast to classical formulations, the bi-Helmholtz nonlocal elasticity model incorporates spatial nonlocal effects alongside temporal memory characteristics inherent in the Green-Naghdi theory, enabling the simultaneous representation of wave-like heat propagation, relaxation behavior, and size-dependent mechanical responses. The introduction of two nonlocal length-scale parameters enhances the model's capability to accurately describe dispersion, attenuation, and thermoelastic coupling, while overcoming the limitations associated with single-phase-lag and single-length-scale approaches.

Recently, combining artificial intelligence methods with traditional thermoelastic and heat transfer models has become a powerful way to improve the ability to make predictions and the speed of calculations in complex multiphysics systems. Conventional analytical and numerical techniques, although precise, frequently incur significant computational costs when addressing strongly coupled, nonlocal, and time-dependent thermoelastic issues, especially within semiconductor materials exposed to laser excitation. This problem gets even harder with more advanced models, like the bi-Helmholtz nonlocal formulation combined with modified Green-Naghdi heat conduction. In these models, multiple field variables, such as displacement, temperature, carrier density, and stress components, are all linked together by higher-order partial differential equations. To overcome these constraints, physics-informed machine learning frameworks, particularly Physics-Informed Neural Networks (PINNs), have garnered considerable interest for their capacity to integrate fundamental physical laws directly into the training methodology. PINNs are different from regular data-driven models because they use soft constraints in the loss function to enforce the underlying differential equations, boundary conditions, and initial conditions. This makes sure that the predictions are physically consistent even when there isn't much data. This feature makes them especially good for thermoelastic semiconductor problems, where keeping the connection between thermal, mechanical, and electronic fields is important for making accurate models. Artificial neural networks (ANN) and gradient boosting algorithms (like XGBoost) are two examples of purely data-driven surrogate models that have been widely used to quickly predict complex physical responses. These models can learn nonlinear connections between input parameters and system responses from datasets that have already been computed. This cuts down on the time it takes to do parametric analysis and optimization studies by a lot. But their performance is often limited by the amount and quality of training data that is available, and they may not be able to be understood in a physical way. In light of these advancements, this study employs a hybrid analytical and artificial intelligence framework to model heat transfer and thermoelastic behavior in a bi-Helmholtz nonlocal semiconductor medium. The analytical solution derived from the normal-mode method functions as a precise reference for generating training data, while a physics-informed neural network is utilized to learn the coupled field responses in a manner consistent with physical principles. Additionally, ANN and XGBoost models are employed as benchmark data-driven predictors to assess the accuracy and efficiency of the proposed methodology. This combined framework not only speeds up calculations, but it also gives you a strong and flexible tool for studying advanced thermoelastic systems that are heated by lasers.

2 Extended Green-Naghdi heat conduction models

The generalization of heat conduction models has evolved from the need to overcome the fundamental limitations of Fourier's law, particularly its prediction of instantaneous thermal signal propagation, which becomes physically unrealistic in transient, microscale, and high-frequency thermal processes [38,39]. Inspired by the strong analogy between viscoelastic mechanics and heat transport, modern theories introduce thermal elastic and thermal viscous elements governed by the concept of thermal displacement as described in Fig. 1, allowing heat transfer to be modeled through combinations of relaxation mechanisms analogous to springs and dashpots. Within this unified framework, classical models such as the Cattaneo–Vernotte and Green–Naghdi theories emerge as specific configurations representing finite-speed thermal waves and non-dissipative or weakly dissipative heat propagation. Further extensions, exemplified by the Modified Green-Naghdi (MGN) hierarchy [39], incorporate multiple relaxation times, higher-order temporal derivatives of heat flux, and temperature-gradient rate effects, thereby capturing memory-dependent behavior, dispersion, attenuation, and transient

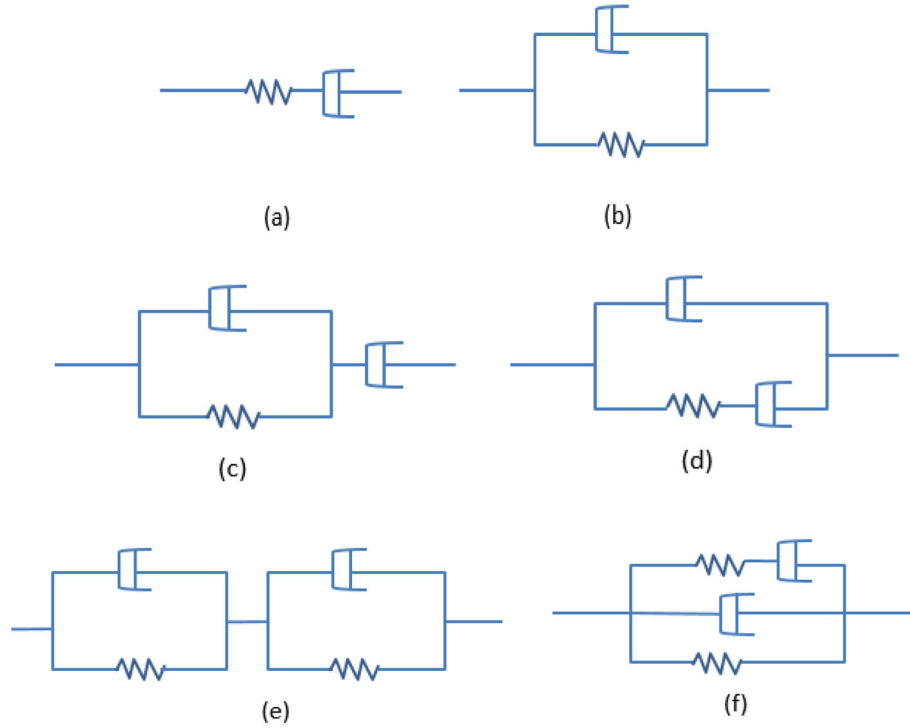


Fig. 1 Representation of generalized modified heat transport models viz. (a) CV, (b) GN, (c) MGN-I, (d) MGN-II, (e) MGN-III, and (f) MGN-IV

overshoot phenomena observed in real materials. These generalized formulations provide a systematic bridge connecting diffusive and wave-dominated heat transport regimes and enable the recovery of several classical theories as limiting cases. Owing to their enhanced physical realism, generalized heat conduction models have become indispensable in the analysis of ultrafast laser–matter interaction, semiconductor and photothermal devices, cryogenic engineering, nanoscale heat transfer, microelectromechanical systems (MEMS), and advanced thermoelastic coupling problems where non-equilibrium thermal effects play a dominant role.

3 Nonlocal kernels of bi-Helmholtz type

Let $\mathcal{V} \subset \mathbb{R}^3$ represent the spatial domain of an infinite elastic medium subjected to external body forces. The fundamental field equations governing the system can be written as

$$t_{ij,j} + \rho f_i = \rho \ddot{u}_i \quad (1)$$

$$t_{ij} = \int_{\mathcal{V}} \alpha(|\mathbf{r} - \mathbf{r}'|) \sigma_{ij}(\mathbf{r}') dv(\mathbf{r}'), \quad t_{ij} = t_{ji} \quad (2)$$

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} \quad (3)$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (4)$$

where λ, μ are Lamé constants, ρ being the density, f_i is the body force per unit mass and u_i being the displacement component. $\alpha(|\mathbf{r} - \mathbf{r}'|)$ is the nonlocal kernel. The boundary conditions involving traction are based on the stress tensor t_{ij} as follows

$$t_{ij} n_j = t_{i(n)}, \quad (5)$$

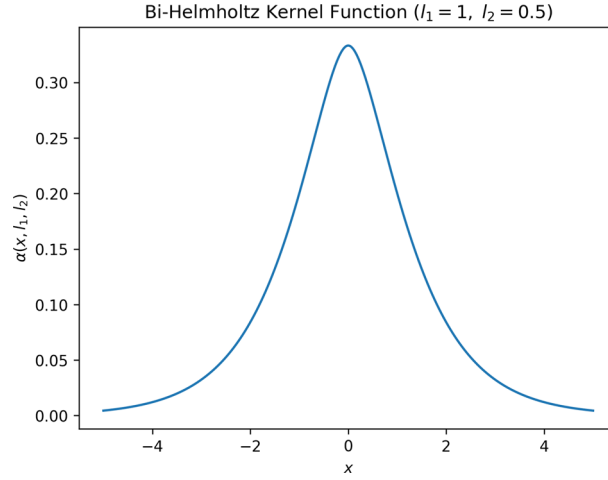


Fig. 2 Representation of bi-Helmholtz type kernel

in which $t_{ij(n)}$ are the traction on the boundary. Invoking the nonlocal stress-strain relation, equation (2) can be represented as

$$t_{ij}(\mathbf{r}) = \int_{\mathcal{V}} \{ \lambda'(|\mathbf{r} - \mathbf{r}'|) \delta_{ij} \delta_{kl} + 2\mu'(|\mathbf{r} - \mathbf{r}'|) \delta_{ij} \delta_{kl} \} e_{kl}(\mathbf{r}') dv(\mathbf{r}'), \quad (6)$$

with

$$\lambda'(|\mathbf{r} - \mathbf{r}'|) = \lambda \alpha(|\mathbf{r} - \mathbf{r}'|), \quad \mu'(|\mathbf{r} - \mathbf{r}'|) = \mu \alpha(|\mathbf{r} - \mathbf{r}'|),$$

which are the Lamé parameters describing the elastic response of the nonlocal medium. The nonlocal kernel represents the spatial coupling function that relates the strain at location $\mathbf{r}'$ to the stress evaluated at $\mathbf{r}$.

However, the nonlocal kernel is a Green's function of a linear differential operator $\mathcal{L}$, satiosfying

$$\mathcal{L}t_{ij} = \sigma_{ij}. \quad (7)$$

The analysis utilizes a fourth-order elliptic differential operator of bi-Helmholtz type introduced by Eringen [40]. This linear operator, possessing constant coefficients, is represented as

$$\mathcal{L} = 1 - \varepsilon^2 \Delta + \gamma^4 \Delta \Delta, \quad (8)$$

where ε and γ denotes the nonnegative parameters justifying nonlocality, possessing the dimension of length and therefore expressed in terms of a characteristic length scale a as

$$\varepsilon = \varepsilon_0 a, \quad \varepsilon_0 \geq 0, \quad \gamma = \gamma_0 a, \quad \gamma_0 \geq 0. \quad (9)$$

The bi-Helmholtz function α is defined by [41,42]

$$\alpha(x, \ell_1, \ell_2) = \frac{1}{2(\ell_1^2 - \ell_2^2)} \left[\ell_1 \exp\left(-\frac{|x|}{\ell_1}\right) - \ell_2 \exp\left(-\frac{|x|}{\ell_2}\right) \right], \quad x \neq 0 \quad (10)$$

$$\alpha(0, \ell_1, \ell_2) = \frac{1}{2(\ell_1^2 + \ell_2^2)}.$$

The graphical representation of the kernel can be illustrated in Fig. 2 as follows:

If ℓ_1 and ℓ_2 are two different nonlocal lenth-scale parameters ($\ell_1 \neq \ell_2$), equation (10) can be rewritten as

$$\alpha(x, \ell_1, \ell_2) = \frac{1}{4\ell_1^2} \left[\ell_1 + |x| \exp\left(-\frac{|x|}{\ell_1}\right) \right], \quad x \neq 0 \quad (11)$$

$$\alpha(0, \ell_1, \ell_2) = \frac{1}{4\ell_1^2}.$$

for $\ell_1 = \ell_2$.

The corresponding fourth-order linear differential operator of bi-Helmholtz type with constant coefficients is represented as [41,42]

$$\Gamma \equiv 1 - (\ell_1^2 + \ell_2^2) \nabla^2 + \ell_1^2 \ell_2^2 \nabla^4. \quad (12)$$

Moreover, it can be shown that

$$\Gamma(x - y, \ell_1, \ell_2) = \delta(x - y), \quad (13)$$

where $\delta(\cdot)$ is the Dirac-delta function.

4 Governing equations

The novel bi-Helmholtz nonlocal theory couples dual-length-scale nonlocal elasticity with the modern non-Fourier heat conduction, enabling a more comprehensive description of photothermal behavior at micro and nanoscale levels. In this context, the constitutive relations for the present problem can be considered as

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{ij} = \mu (u_{i,j} + u_{j,i}) + (\lambda e_{kk} - \gamma_n N - \gamma_t T) \delta_{ij}. \quad (14)$$

The equation of motion in absence of body forces are given by

$$\sigma_{ij,j} = \rho \ddot{u}_i. \quad (15)$$

The equation of carrier density is given by

$$\frac{\partial N}{\partial t} = D_E N_{,kk} - \frac{N}{\tau_1} + \kappa T. \quad (16)$$

When a flexible semiconductor medium is subjected to external optical irradiation, incident photons excite electrons across the semiconductor band gap energy E_g , generating nonequilibrium free carriers and producing spatial variations in carrier density. This photoinduced carrier generation initiates coupled mechanical responses, leading to deformation and the propagation of elastic waves within the medium. Concurrently, the interaction among thermal effects, elastic deformation, and plasma dynamics significantly alters the classical heat transport mechanism. The absorbed optical energy is closely associated with the electron-hole recombination process, which acts as an internal heat source governing energy redistribution. As a consequence, the classical Fourier heat conduction law becomes insufficient to describe heat transfer in such photoexcited semiconductor environments. Therefore, a modified form of Fourier's law is introduced by incorporating unified parameters that account for the coupled photothermal, thermoelastic, and plasma transport phenomena as follows

$$-K \nabla T - K \chi_1 \tau_0 \nabla \dot{T} - \chi_2 K \tau_0^2 \nabla \ddot{T} - \int \frac{E_g}{\tau_1} N d\mathbf{r} = (\chi_3 \tau_0 + \chi_4 \tau_0^2 D_t) \dot{q}. \quad (17)$$

Therefore, the equation for heat transport yields

$$K \nabla^2 T + K \chi_1 \tau_0 \nabla^2 \dot{T} + K \chi_2 \tau_0^2 \nabla^2 \ddot{T} + \frac{E_g}{\tau_1} \dot{N} = \left(\chi_3 \tau_0 + \chi_4 \tau_0^2 \frac{\partial}{\partial t} \right) [\rho c_E \ddot{T} + \gamma_t T_0 \ddot{e}]. \quad (18)$$

In the heat transport formulation, χ_i ($i = 1, 2, 3, 4$) correspond to unified parameters that generalize different heat conduction models under a single theoretical framework. As special case

$$\begin{aligned} \chi_1 = 1, \quad \chi_2 = 0, \quad \chi_3 = 1, \quad \chi_4 = 0 & \text{ corresponds to GN model,} \\ \chi_1 = 1, \quad \chi_2 = 0, \quad \chi_3 = 2, \quad \chi_4 = 1 & \text{ corresponds to MGN-I model,} \\ \chi_1 = 1, \quad \chi_2 = 2, \quad \chi_3 = 1, \quad \chi_4 = 1 & \text{ corresponds to MGN-II model,} \\ \chi_1 = 1, \quad \chi_2 = 1, \quad \chi_3 = 2, \quad \chi_4 = 2 & \text{ corresponds to MGN-III model,} \\ \chi_1 = 3, \quad \chi_2 = 1, \quad \chi_3 = 1, \quad \chi_4 = 1 & \text{ corresponds to MGN-IV model.} \end{aligned}$$

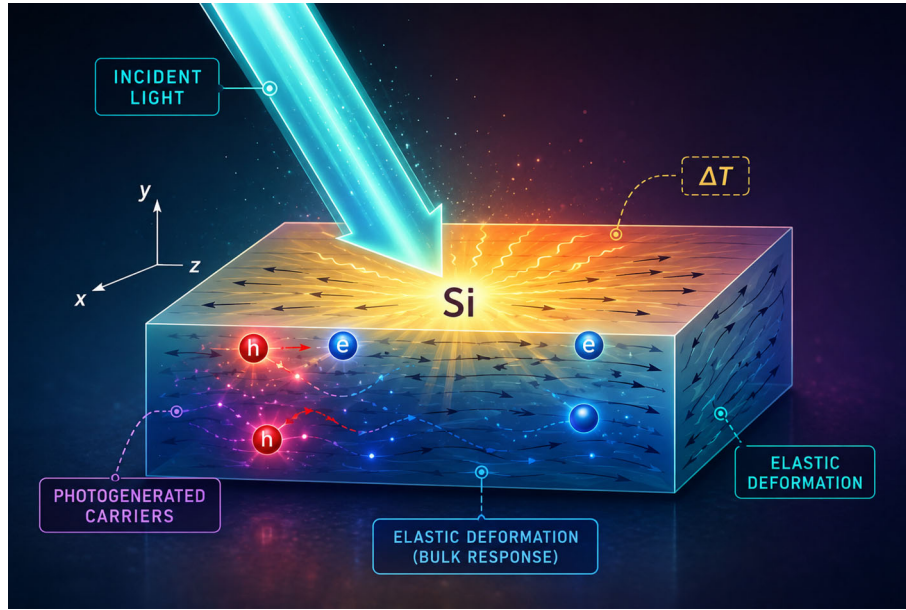


Fig. 3 Schematic diagram of the opto-electronic semiconductor

The carrier continuity equation is obtained from the coupling between thermal and plasma waves during the photoexcitation process in the following form [43–45]

$$\frac{\partial N}{\partial t} = D_E \nabla^2 N - \frac{N}{\tau_1} + \kappa T. \quad (19)$$

In the presence of optical energy absorption, the coupled interaction between thermal and plasma waves is regulated by the thermal activation coupling parameter, reflecting the degree of energy exchange between thermal and carrier dynamics as $\kappa = \frac{\partial N_0}{\partial T} \frac{T}{\tau_1}$, n_0 being the equilibrium carrier concentration.

5 Problem formulation

We investigate a two-dimensional homogeneous and isotropic silicon semiconductor subjected to opto-electronic excitation, as depicted in Fig. 3. The material parameters are assumed to remain constant throughout the medium, ensuring spatial uniformity. Hence, the analysis is limited to the xy -plane, and all thermophysical quantities are considered functions of x , y and time t only. Consequently, the displacement components and temperature field are expressed as follows

$$\mathbf{u} = (u(x, y, t), v(x, y, t), 0), \quad T = T(x, y, t). \quad (20)$$

Adopting the bi-helmholtz nonlocal effects, the constitutive equations yield

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \gamma_n N - \gamma_t T, \quad (21)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} - \gamma_n N - \gamma_t T, \quad (22)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). \quad (23)$$

The equations of motion are given by

$$(\lambda + \mu) \frac{\partial e}{\partial x} + \mu \nabla^2 u - \gamma_n \frac{\partial N}{\partial x} - \gamma_t \frac{\partial T}{\partial x} = \rho (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \ddot{u}, \quad (24)$$

$$(\lambda + \mu) \frac{\partial e}{\partial y} + \mu \nabla^2 v - \gamma_n \frac{\partial N}{\partial y} - \gamma_t \frac{\partial T}{\partial y} = \rho (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \ddot{v}. \quad (25)$$

The heat conduction equation is given by

$$K \nabla^2 T + K \chi_1 \tau_0 \nabla^2 \dot{T} + K \chi_2 \tau_0^2 \nabla^2 \ddot{T} + \frac{E_g}{\tau_1} \dot{N} = \left(\chi_3 \tau_0 + \chi_4 \tau_0^2 \frac{\partial}{\partial t} \right) [\rho c_E \ddot{T} + \gamma_t T_0 \ddot{e}]. \quad (26)$$

Invoking the nondimensional variables

$$x^* = c_0 \eta x, \quad y^* = c_0 \eta y, \quad u^* = c_0 \eta u, \quad v^* = c_0 \eta v, \quad \ell_1^* = c_0 \eta \ell_1, \quad \ell_2^* = c_0 \eta \ell_2, \quad t^* = c_0^2 \eta t,$$

$$\tau_1^* = c_0^2 \eta \tau_1, \quad T^* = \frac{\gamma_t T}{\lambda + 2\mu}, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\mu}, \quad N^* = \frac{\gamma_n N}{\lambda + 2\mu}, \quad \eta = \frac{\rho c_E}{K}, \quad c_0 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

and after discarding asterisk, the nondimensional expressions of the aforesaid equations can be expressed as

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xx} = \beta^2 \frac{\partial u}{\partial x} + (\beta^2 - 2) \frac{\partial v}{\partial y} - \beta^2 N - \beta^2 T, \quad (27)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{yy} = \beta^2 \frac{\partial v}{\partial y} + (\beta^2 - 2) \frac{\partial u}{\partial x} - \beta^2 N - \beta^2 T, \quad (28)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (29)$$

$$\left(1 - \frac{1}{\beta^2} \right) \frac{\partial e}{\partial x} + \frac{1}{\beta^2} \nabla^2 u - \frac{\partial N}{\partial x} - \frac{\partial T}{\partial x} = (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \ddot{u}, \quad (30)$$

$$\left(1 - \frac{1}{\beta^2} \right) \frac{\partial e}{\partial y} + \frac{1}{\beta^2} \nabla^2 v - \frac{\partial N}{\partial y} - \frac{\partial T}{\partial y} = (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \ddot{v}, \quad (31)$$

$$\nabla^2 T + \chi_1 \tau_0 \nabla^2 \dot{T} + \chi_2 \tau_0^2 \nabla^2 \ddot{T} + \epsilon_1 \dot{N} = \left(\chi_3 \tau_0 + \chi_4 \tau_0^2 \frac{\partial}{\partial t} \right) [\ddot{T} + \epsilon_T \ddot{e}], \quad (32)$$

$$\epsilon_2 \frac{\partial N}{\partial t} = \nabla^2 N - \epsilon_2 \frac{N}{\tau_1} + \epsilon_3 T, \quad (33)$$

where

$$\beta^2 = \frac{\lambda + 2\mu}{\mu}, \quad \epsilon_1 = \frac{\gamma_t c_0^2 \eta}{\gamma_n \rho c_E \tau_1}, \quad \epsilon_2 = \frac{1}{D_E \eta}, \quad \epsilon_3 = \frac{\kappa \gamma_n}{D_E \gamma_t c_0^2 \eta^2}, \quad \epsilon_T = \frac{\gamma_t^2 T_0}{\rho c_E (\lambda + 2\mu)}.$$

The scalar and vector potentials q and $\vec{U}$ are defined via the Helmholtz decomposition theorem as follows:

$$\vec{u} = \nabla q + \nabla \times \vec{U}, \quad \nabla \cdot \vec{U} = 0, \quad (34)$$

where, the scalar potential $q(x, y, t)$ and vector potential $\vec{U}(x, y, t)$ characterize the irrotational and rotational parts of the displacement field, respectively. Hence, the displacement components u and v can be written as

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial q}{\partial y} - \frac{\partial \psi}{\partial x} \quad \text{and} \quad \psi = (-\vec{U})_y \quad (35)$$

then the above equations reduce to

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xx} = \beta^2 \nabla^2 q - 2 \frac{\partial^2 q}{\partial y^2} + 2 \frac{\partial^2 \psi}{\partial x \partial y} - \beta^2 N - \beta^2 T, \quad (36)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{yy} = \beta^2 \nabla^2 q - 2 \frac{\partial^2 q}{\partial x^2} - 2 \frac{\partial^2 \psi}{\partial x \partial y} - \beta^2 N - \beta^2 T, \quad (37)$$

$$(1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \sigma_{xy} = 2 \frac{\partial^2 q}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}, \quad (38)$$

$$\nabla^2 q - N - T = (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \frac{\partial^2 q}{\partial t^2}, \quad (39)$$

$$\frac{1}{\beta^2} \nabla^2 \psi - (1 - \ell_1^2 \nabla^2) (1 - \ell_2^2 \nabla^2) \frac{\partial^2 \psi}{\partial t^2} = 0, \quad (40)$$

$$\nabla^2 T + \chi_1 \tau_0 \nabla^2 \dot{T} + \chi_2 \tau_0^2 \nabla^2 \ddot{T} + \epsilon_1 \dot{N} = \left(\chi_3 \tau_0 + \chi_4 \tau_0^2 \frac{\partial}{\partial t} \right) [\ddot{T} + \epsilon_T \ddot{q}], \quad (41)$$

$$\epsilon_2 \frac{\partial N}{\partial t} = \nabla^2 N - \epsilon_2 \frac{N}{\tau_1} + \epsilon_3 T. \quad (42)$$

6 Normal mode analysis

To analyze the wave propagation characteristics, the physical quantities are expanded in terms of normal modes, taking the following form:

$$(q, \psi, T, \sigma_{ij}, N)(x, y, t) = (q^*, \psi^*, T^*, \sigma_{ij}^*, N^*)(x) e^{\omega t + \iota b y}, \quad (43)$$

where ω (complex) is the frequency, b being the wave number in y -direction and $q^*, \psi^*, T^*, \sigma_{ij}^*, N^*$ are the amplitudes of the respective physical quantities and $\iota = \sqrt{-1}$ being the imaginary number. Recalling the relationship of normal modes defined in the relation (43), thereafter equations (36)–(42) can be respectively expressed as

$$(\eta_1 - \ell_1^2 D^2) (\eta_2 - \ell_2^2 D^2) \sigma_{xx}^* = \beta^2 D^2 q^* + (2 - \beta^2) b^2 q^* + 2\iota b D \psi^* - \beta^2 (N^* + T^*), \quad (44)$$

$$(\eta_1 - \ell_1^2 D^2) (\eta_2 - \ell_2^2 D^2) \sigma_{yy}^* = (\beta^2 - 2) D^2 q^* - \beta^2 b^2 q^* - 2\iota b D \psi^* - \beta^2 (N^* + T^*), \quad (45)$$

$$(\eta_1 - \ell_1^2 D^2) (\eta_2 - \ell_2^2 D^2) \sigma_{xy}^* = 2\iota b D q^* - (D^2 + b^2) \psi^*, \quad (46)$$

$$c_{11} D^4 q^* - c_{12} D^2 q^* + c_{13} q^* + N^* + T^* = 0, \quad (47)$$

$$D^4 \psi^* - c_{31} D^2 \psi^* + c_{32} \psi^* = 0, \quad (48)$$

$$[D^2 - (b^2 + \xi_1)] T^* + \xi_2 N^* = \xi_1 \epsilon_T (D^2 - b^2) q^*, \quad (49)$$

$$(D^2 - \xi_3) N^* = -\epsilon_3 T^*. \quad (50)$$

From equations (47), (49) and (50), eliminating T^* , consequently, the governing differential equations for N^* and q^* takes the form

$$(D^8 - \mathcal{N}_1 D^6 + \mathcal{N}_2 D^4 - \mathcal{N}_3 D^2 + \mathcal{N}_4) (N^*, q^*) = 0, \quad (51)$$

where

$$\mathcal{N}_1 = \frac{1}{c_{11}} \left[c_{41} c_{11} + c_{12} \right],$$

$$\mathcal{N}_2 = \frac{1}{c_{11}} \left[c_{11} c_{42} + c_{12} c_{41} + c_{12} + \xi_1 \epsilon_T \right],$$

$$\mathcal{N}_3 = \frac{1}{c_{11}} \left[c_{12} c_{42} + c_{13} c_{41} + \xi_1 \epsilon_T (\xi_3 + \epsilon_3 + b^2) \right],$$

$$\mathcal{N}_4 = \frac{1}{c_{11}} \left[c_{13} c_{42} + b^2 \xi_1 \epsilon_T (\xi_3 + \epsilon_3) \right],$$

$$\eta_1 = 1 + \ell_1^2 b^2, \quad c_{11} = \ell_1^2 \ell_2^2 \omega^2,$$

$$\eta_2 = 1 + \ell_2^2 b^2, \quad c_{12} = \omega^2 (\eta_2 \ell_1^2 + \eta_1 \ell_2^2) + 1,$$

$$\begin{aligned}\xi_1 &= \frac{\omega^2 (\chi_3 \tau_0 + \chi_4 \tau_0^2 \omega)}{1 + \chi_1 \tau_0 \omega + \chi_2 \tau_0^2 \omega^2}, & c_{13} &= \eta_1 \eta_2 \omega^2 + b^2, \\ \xi_2 &= \frac{\epsilon_1 \omega}{1 + \chi_1 \tau_0 \omega + \chi_2 \tau_0^2 \omega^2}, & \xi_3 &= b^2 + \frac{\epsilon_2}{\tau_1} + \epsilon_2 \omega, \\ c_{31} &= \frac{1}{c_{11}} \left\{ c_{12} + \frac{1}{\beta^2} - 1 \right\}, & c_{32} &= \frac{1}{c_{11}} \left\{ c_{13} + b^2 \left(1 - \frac{1}{\beta^2} \right) \right\}, \\ c_{41} &= b^2 + \xi_1 + \xi_3, & c_{42} &= \xi_3 (\xi_1 + b^2) - \xi_2 \epsilon_3.\end{aligned}$$

The analytical solution associated with the differential equation (51) can be represented as:

$$N^* = \sum_{j=1}^4 \mathcal{A}_j e^{-k_j x}, \quad (52)$$

$$q^* = \sum_{j=1}^4 \mathcal{B}_j e^{-k_j x}, \quad (53)$$

where k_j^2 are roots of the characteristic equation

$$k^8 - \mathcal{N}_1 k^6 + \mathcal{N}_2 k^4 - \mathcal{N}_3 k^2 + \mathcal{N}_4 = 0. \quad (54)$$

Substituting the solutions (52) and (53) in the simplified form of equations (49) and (50), we have

$$q^* = \sum_{j=1}^4 h_{1j} \mathcal{A}_j e^{-k_j x}, \quad (55)$$

where

$$h_{1j} = \frac{k_j^4 - c_{41} k_j^2 + c_{42}}{\xi_1 \epsilon_3 \epsilon_T (b^2 - k_j^2)}.$$

Accordingly, substituting the obtained solution (52) into equation (50) results in the temperature distribution of the semiconductor medium given by:

$$T^* = \frac{1}{\epsilon_3} \sum_{j=1}^4 \left(\xi_3 - k_j^2 \right) \mathcal{A}_j e^{-k_j x}, \quad (56)$$

Further, from equation (48), the solution for the potential function ψ^* can be illustrated as

$$\psi^* = \sum_{j=1}^2 \mathcal{C}_j e^{-p_j x}, \quad (57)$$

where p_1^2, p_2^2 are roots of the characteristic equation

$$k^4 - c_{31} k^2 + c_{32} = 0. \quad (58)$$

7 Physics-Informed Neural Network (PINN) Framework

7.1 Motivation for AI Integration

The analytical solution offered in this study delivers a precise characterization of the coupled thermoelastic response in a bi-Helmholtz nonlocal semiconductor medium subjected to opto-electronic excitation. But because the governing coupled partial differential equations are so complicated, the cost of running the same tests over and over for parametric studies, optimization, and real-time prediction can be very high.

To get around this problem, a physics-informed neural network (PINN) framework is introduced to efficiently approximate the solution fields while keeping the basic physical laws intact. PINNs are different from other data-driven models because they put the governing equations right into the learning process. This lets them make accurate predictions even when there isn't much training data. This makes them especially good for multiphysics problems where thermal, mechanical, and carrier transport phenomena are strongly linked.

7.2 PINN formulation

Let, the neural network approximate the solution fields as

$$(u, v, T, N) = \mathcal{N}_\theta(x, y, t), \quad (59)$$

where $\mathcal{N}_\theta$ represents a neural network with parameter θ having inputs (x, y, t) and the respective outputs are the displacement components u, v , temperature T and the carrier density N .

The governing equations of the bi-Helmholtz nonlocal thermoelastic model are enforced through automatic differentiation as

$$\mathcal{R}_1 = (1 - \ell^2 \nabla^2) \nabla^2 u - \beta_1 \nabla T = 0, \quad (60)$$

$$\mathcal{R}_2 = (1 - \ell^2 \nabla^2) \nabla^2 v - \beta_1 \nabla T = 0, \quad (61)$$

$$\mathcal{R}_3 = \rho c_1 (1 - \ell_1^2 \nabla^2) T + (\chi_1 \dot{T} + \tau_1 T) = 0, \quad (62)$$

$$\mathcal{R}_4 = (1 - \ell_2^2 \nabla^2) N - f(x, y, t) = 0. \quad (63)$$

These residuals represent the physics constraints embedded in the neural network.

8 Loss function construction

The total loss function is defined as

$$\mathcal{L} = \mathcal{L}_{PDE} + \mathcal{L}_{BC} + \mathcal{L}_{IC} + \mathcal{L}_{data}, \quad (64)$$

where, the PDE loss is represented by

$$\mathcal{L}_{PDE} = \sum_{i=1}^4 \|\mathcal{R}_i\|^2. \quad (65)$$

The loss function of the boundary conditions yields

$$\mathcal{L}_{BC} = \|u - 0\|^2 + \|v - 0\|^2 + \|\sigma_{xx}\|^2 + \left\| \frac{\partial \sigma_{xx}}{\partial x} \right\|^2 + \|T - T_0\|^2. \quad (66)$$

The loss function corresponding to the initial condition is given by

$$\mathcal{L}_{IC} = \|u(x, y, 0)\|^2 + \|v(x, y, 0)\|^2 + \|T(x, y, 0)\|^2 + \|N(x, y, 0)\|^2. \quad (67)$$

Moreover, the data loss function is given by

$$\mathcal{L}_{data} = \|u - u^{analytical}\|^2 + \|T - T^{analytical}\|^2 + \dots \quad (68)$$

8.1 Training strategy and implementation

The neural network is trained using collocation points sampled in the spatial-temporal domain (x, y, t) . Automatic differentiation is employed to compute spatial and temporal derivatives required in the governing equations. The model is implemented using a fully connected deep neural network architecture, which enables effective approximation of the underlying physical fields. The hyperbolic tangent (tanh) activation function is employed to introduce nonlinearity and ensure smooth learning behavior across the network. For optimization, a two-stage strategy is adopted: initially, the Adam optimizer is used to achieve rapid convergence, followed by the L-BFGS algorithm for fine-tuning and improved accuracy. The training dataset is generated from the corresponding analytical solution, ensuring reliability and consistency in the learning process. Model performance is evaluated using standard metrics, including mean absolute error (MAE), root mean square error (RMSE), and the coefficient of determination (R^2). The overall training procedure is designed to minimize a total loss function, which enforces that the predicted fields not only fit the data but also satisfy the governing equations and prescribed boundary conditions.

8.2 Data-Driven Benchmark Models (ANN & XGBoost)

To assess the performance of the PINN model, additional data-driven surrogate models are implemented using artificial neural networks (ANN) and extreme gradient boosting (XGBoost). IN this process, the input features include $(x, y, t, \ell_1, \ell_2, \tau_0)$ and the outputs correspond to $(u, v, T, N, \sigma_{xx}, \sigma_{xy})$. These models are trained using datasets generated from the analytical solution. While ANN and XGBoost provide fast predictions, they do not explicitly enforce physical constraints, unlike PINNs.

9 Boundary conditions

(i) Displacement boundary conditions

The illuminated surface is rigidly fixed in the longitudinal and transverse directions. This yields

$$u(0, y, t) = 0, \quad v(0, y, t) = 0. \quad (69)$$

(ii) Stress boundary conditions At the photo-induced illuminated surface, the normal stress disappears across the surface. This yields

$$\sigma_{xx}(0, y, t) = 0. \quad (70)$$

Moreover, it is assumed that on the boundary, no gradient traction is applied. This yields

$$\frac{\partial \sigma_{xx}(0, y, t)}{\partial x} = 0. \quad (71)$$

(iii) Thermal boundary condition

The illuminated surface is influenced by a prescribed temperature

$$T(0, y, t) = T_0. \quad (72)$$

(iv) Mechanical boundary condition

Taking into account the diffusion and recombination processes of photogenerated carriers, the corresponding boundary condition for the carrier density may be written as:

$$D_E \frac{\partial N(0, y, t)}{\partial x} = S_s N(0, y, t), \quad (73)$$

where S_s is the recombination velocity associated with the bounding plane and serves as a measure of the recombination activity at the surface. When surface recombination is absent, the motion of carriers toward the boundary vanishes.

However, in terms of normal modes, the displacement of the photothermoelastic medium is given by

$$u = \left[ib \sum_{j=1}^2 C_j e^{-p_j x} - \sum_{j=1}^4 h_{1j} k_j A_j e^{-k_j x} \right] e^{\omega t + i b y}, \quad (74)$$

$$v = \left[ib \sum_{j=1}^4 h_{1j} A_j e^{-k_j x} + \sum_{j=1}^2 C_j p_j e^{-p_j x} \right] e^{\omega t + i b y}. \quad (75)$$

Moreover, the amplitudes of the normal stress and the shearing stress can be illustrated invoking equations (44) and (45) as follows

$$\sigma_{xx}^* = \sum_{j=1}^4 \frac{\Phi(k_j)}{(\eta_1 - \ell_1^2 k_j^2)(\eta_2 - \ell_2^2 k_j^2)} A_j e^{-k_j x} - 2ib \sum_{j=1}^2 \frac{C_j p_j}{(\eta_1 - \ell_1^2 p_j^2)(\eta_2 - \ell_2^2 p_j^2)} e^{-p_j x}, \quad (76)$$

$$\sigma_{xy}^* = -2ib \sum_{j=1}^4 \frac{h_{1j} k_j}{(\eta_1 - \ell_1^2 k_j^2)(\eta_2 - \ell_2^2 k_j^2)} A_j e^{-k_j x} - \sum_{j=1}^2 \frac{(p_j^2 + b^2)}{(\eta_1 - \ell_1^2 p_j^2)(\eta_2 - \ell_2^2 p_j^2)} C_j e^{-p_j x}, \quad (77)$$

where

$$\Phi(k_j) = \beta^2 h_{1j} (k_j^2 - b^2) + 2b^2 h_{1j} - \beta^2 \left(1 + \frac{\xi_3 - k_j^2}{\epsilon_3} \right).$$

Finally, on using the dimensionless form of the boundary conditions (69)-(73), we arrive at the system of equations in the following form as follows

$$\begin{aligned} ib \sum_{j=1}^2 C_j - \sum_{j=1}^4 h_{1j} k_j A_j &= 0, \\ ib \sum_{j=1}^4 h_{1j} A_j + \sum_{j=1}^2 C_j p_j &= 0, \\ \sum_{j=1}^4 \frac{\Phi(k_j)}{(\eta_1 - \ell_1^2 k_j^2)(\eta_2 - \ell_2^2 k_j^2)} A_j - 2ib \sum_{j=1}^2 \frac{C_j p_j}{(\eta_1 - \ell_1^2 p_j^2)(\eta_2 - \ell_2^2 p_j^2)} &= 0 \\ \sum_{j=1}^4 \frac{\Phi(k_j) k_j}{(\eta_1 - \ell_1^2 k_j^2)(\eta_2 - \ell_2^2 k_j^2)} A_j - 2ib \sum_{j=1}^2 \frac{C_j p_j^2}{(\eta_1 - \ell_1^2 p_j^2)(\eta_2 - \ell_2^2 p_j^2)} &= 0 \\ \frac{1}{\epsilon_3} \sum_{j=1}^4 (\xi_3 - k_j^2) A_j &= T_0, \\ -D_E \sum_{j=1}^4 A_j k_j &= S_s \sum_{j=1}^4 A_j. \end{aligned} \quad (78)$$

Upon solving the above system of equations (78), the unknown parameters A_j ($j = 1(1)4$), C_j , $j = 1, 2$ respectively will be determined. This completes the solution of the present problem.

10 Model assessment and validation

To ensure the reliability of the proposed model, the obtained results are compared with earlier established results listed in Table 1. As illustrated in Fig. 10, when spatiotemporal nonlocal effects are neglected, the present numerical outcomes show excellent agreement with the results reported in [47], confirming the numerical validity of the formulation under GN theory.

Table 1 Case study of associated models and selected cases

Case	Condition	Model
I	$\ell = 0, \tau = 0, \chi_i = 0 \ (i = 1(1)4)$	For Case I, excluding spatiotemporal nonlocal effects, the MGN model predictions closely reproduce the results of [47], thereby confirming the correctness and numerical reliability of the formulation within the Green–Naghdi framework.
II	$\ell = 0, \tau = 0, \tau_0 = 0$	Case II demonstrates excellent agreement between the present results and those reported in [48] when both spatiotemporal nonlocal effects and temperature dependence of the elastic parameters are neglected. Under these assumptions, the formulation reduces to the classical Green–Naghdi (GN) thermoelastic model, thereby validating the accuracy and consistency of the present analysis.

11 Numerical results and interpretations

With an aim to illustrate the solution of the thermophysical quantities such as the temperature, displacement, carrier density, stress, excess pore water pressure and photoacoustic pressure in the space-time domain, we now implement suitable numerical scheme for the determination of the solution. To demonstrate the numerical results, silicon (Si) is chosen as the reference material, and the required material constants are obtained from [46,47]:

$$\begin{aligned}
 \lambda &= 6.4 \times 10^{10} \text{ N m}^{-2}, \quad \mu = 6.5 \times 10^{10} \text{ N m}^{-2}, \quad \alpha_t = 2.6 \times 10^{-6} \text{ K}^{-1}, \\
 K &= 150 \text{ W m}^{-1} \text{ K}^{-1}, \quad c_E = 695 \text{ J kg}^{-1} \text{ K}^{-1}, \quad \rho = 2330 \text{ kg m}^{-3}, \\
 T_0 &= 800 \text{ K}, \quad \tau_1 = 5 \times 10^{-5} \text{ s}, \quad D_E = 2.5 \times 10^3 \text{ kg m}^{-3}, \\
 E_g &= 1.11 \text{ eV}, \quad d_n = -9 \times 10^{-31} \text{ m}^3, \quad \mathcal{N}_0 = 1, \\
 T_0 &= 2, \quad \zeta_0 = 1, \quad S_s = 2 \text{ ms}^{-1}
 \end{aligned}$$

11.1 Implications of nonlocal length scales parameters

A detailed discussion on the impact of nonlocal parameters on thermophysical field characteristics is presented in this subsection, as shown in Fig. 4(a)–(f). All these numerical computations have been carried out at depth $y = 0.1$ at instant $t = 0.15$, while the other parameters have been considered as $\tau_0 = 0.02$, $b = 1$, $\chi_1 = 3$, $\chi_2 = 1$, $\chi_3 = 1$, and $\chi_4 = 1$ i.e., the heat conduction equation of the present problem is framed in the context of MGN-IV model.

Figure 4(a) illustrates the spatial variation of the horizontal displacement u with respect to the distance x at the fixed instant $t = 0.15$, incorporating the effects of Bi-Helmholtz spatial nonlocality. The displacement field initiates from zero at the boundary $x = 0$, thereby satisfying the prescribed clamped boundary condition. In the immediate vicinity of the illuminated surface, the displacement increases and attains a maximum around $x \approx 0.2$. This localized peak arises due to the strong thermoelastic coupling induced by optical excitation and the associated carrier–lattice interaction near the surface. Beyond this region, a rapid attenuation in magnitude is observed, reflecting the dissipative and dispersive nature of wave propagation in the nonlocal medium. Furthermore, increasing the spatial nonlocal parameters enhances the magnitude and spatial spread of the displacement field, indicating that intrinsic length-scale effects significantly influence mechanical deformation in the semiconductor.

Figure 4(b) presents the corresponding variation of the vertical displacement v . Similar to the horizontal component, the displacement satisfies the essential boundary condition by vanishing at $x = 0$. A monotonic rise in magnitude occurs within the near-boundary region ($0 < x < 0.2$), followed by a pronounced decay after reaching its peak value. This behavior reflects the combined influence of elastic wave dispersion and thermal relaxation inherent in the modified Green–Naghdi formulation. The qualitative effect of increasing nonlocal parameters remains consistent: larger length-scale parameters modify both amplitude and decay characteristics, demonstrating the sensitivity of transverse motion to spatial nonlocality.

Figure 4(c) depicts the temperature distribution along the depth direction for different selections of nonlocal length-scale parameters. The temperature profile originates from the prescribed thermal loading at $x = 0$, thus

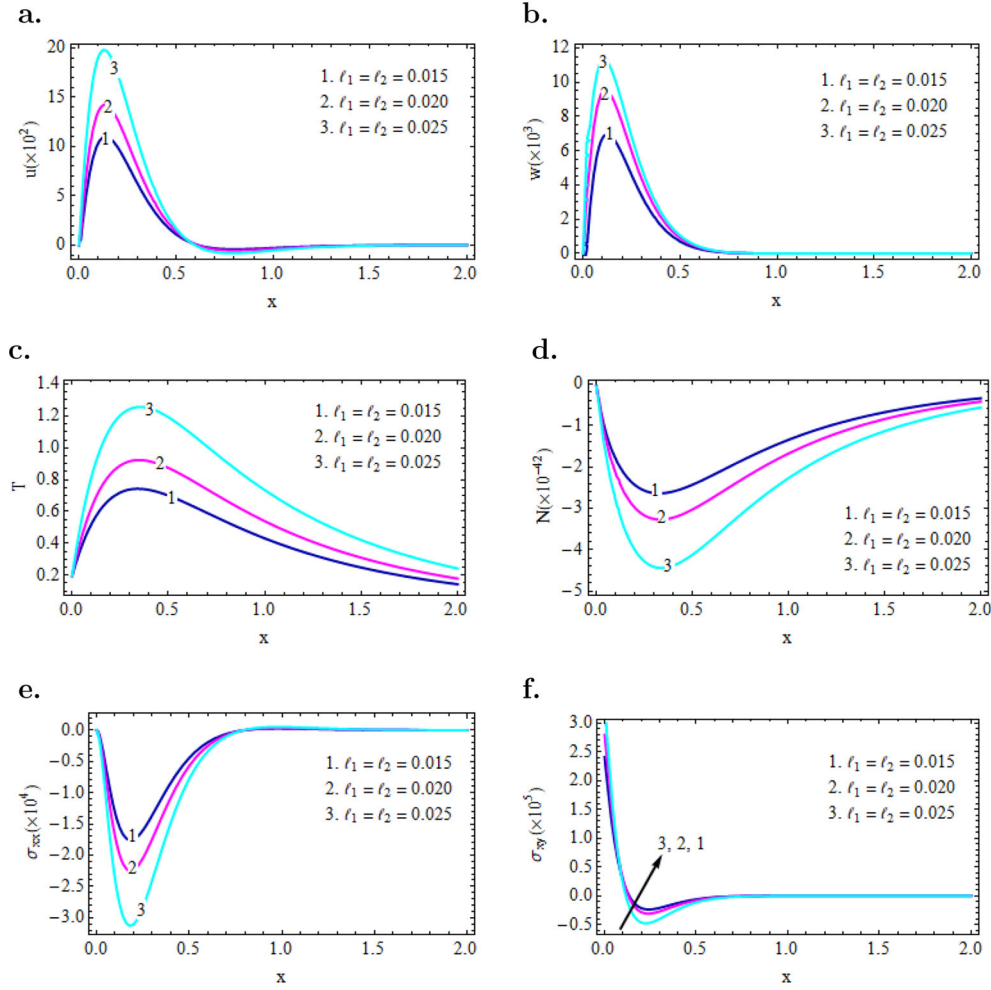


Fig. 4 Distributions of **a.** horizontal displacement u , **b.** vertical displacement w , **c.** temperature T , **d.** carrier density N , **e.** normal stress σ_{xx} and **f.** shear stress σ_{xy} vs. x for different nonlocality parameters ℓ_1 and ℓ_2

validating the imposed Dirichlet thermal boundary condition. A slight increase in temperature is observed near the boundary due to localized photothermal absorption and carrier recombination processes, which act as internal heat sources. However, as the distance from the illuminated surface increases, the temperature decreases rapidly, signifying thermal diffusion and wave-type heat transport predicted by the modified Green–Naghdi theory. The decay rate becomes more pronounced with increasing nonlocal parameters ℓ_1 and ℓ_2 , highlighting the role of the bi-Helmholtz kernel in altering thermal wave dispersion and attenuation characteristics.

Figure 4(d) demonstrates the distribution of carrier density N within the semiconductor. The carrier density vanishes at the boundary, consistent with the imposed surface recombination condition. A compressive-type spatial profile is observed, where the carrier concentration increases within a finite penetration depth before diminishing toward zero at larger distances. This behavior is governed by diffusion–recombination balance and the coupling between carrier dynamics and the thermoelastic field. The influence of the nonlocal parameters modifies the penetration depth and peak concentration, emphasizing the interplay between microstructural length scales and carrier transport.

Figure 4(e) and (f) present the variations of the normal stress σ_{xx} and shear stress σ_{xy} , respectively. The normal stress satisfies the mechanical boundary condition by vanishing at $x = 0$. A predominantly compressive stress state develops within the medium, which can be attributed to thermal expansion constrained by the boundary and photogenerated carrier pressure. The stress magnitude decays with increasing depth, reflecting the attenuation of coupled thermoelastic waves. The shear stress exhibits a similar qualitative trend, governed by transverse deformation and mode coupling effects. In both cases, the nonlocal parameters significantly

influence stress amplitude and spatial distribution, confirming that the bi-Helmholtz formulation effectively captures size-dependent mechanical responses absent in classical local theories.

Overall, the results clearly demonstrate that the incorporation of Bi-Helmholtz spatial nonlocality modifies displacement, temperature, carrier density, and stress fields in a physically consistent manner, highlighting the importance of intrinsic material length scales in opto-electronic semiconductor applications.

11.2 Implications of thermal phase lag parameter

The present subsection is addressed to highlight the influence of the relaxation-time parameter τ_0 on the behavior of the thermophysical fields. For the purpose of computations, the heat transport law is designed in the context of MGN-IV heat conduction law, while the parameters of the computation have been considered to be $\ell_1 = \ell_2 = 0.01$, $t = 0.15$, $y = 0.1$, $b = 1$, $\chi_1 = 3$, $\chi_2 = 1$, $\chi_3 = 1$, and $\chi_4 = 1$.

As noticed from Fig. 5(a), the horizontal displacement profile demonstrates a pronounced sensitivity to the relaxation time. For smaller values of τ_0 , the displacement amplitude is significantly higher, exhibiting a sharp peak near the illuminated surface. As τ_0 increases, the peak magnitude decreases and the profile becomes smoother. Physically, the relaxation time governs the delay in heat flux response to temperature gradients. A larger τ_0 introduces stronger thermal inertia, reducing the immediate thermoelastic coupling strength. Consequently, mechanical deformation becomes less intense as the relaxation effect strengthens.

A similar qualitative trend is observed in the transverse displacement component as revealed in Fig. 5(b). For lower τ_0 , the amplitude is larger and more localized near the boundary. Increasing τ_0 suppresses the peak magnitude and accelerates spatial attenuation. This indicates that relaxation effects reduce the effective thermoelastic driving force responsible for transverse motion, introducing a damping-like behavior in the coupled system.

From Fig. 5(c), the temperature field shows moderate yet systematic dependence on τ_0 . As the relaxation time increases, the peak temperature slightly decreases and the decay becomes smoother. Under the MGN-IV model, heat propagates as a thermal wave with finite speed rather than diffusing instantaneously. A larger τ_0 increases the delay in heat flux establishment, thereby distributing thermal energy more gradually within the medium. This leads to reduced thermal gradients and weaker thermoelastic feedback.

In Fig. 5(d), the carrier density profile exhibits noticeable dependence on τ_0 . With increasing relaxation time, the magnitude of the carrier concentration decreases slightly and the spatial penetration depth is modified. Since carrier transport is strongly coupled with temperature through diffusion and recombination mechanisms, the relaxation-induced modification of thermal gradients indirectly influences carrier dynamics.

In Fig. 5(e), the normal stress shows a compressive character near the boundary. For smaller τ_0 , the compressive peak is stronger and more localized. As τ_0 increases, the stress magnitude decreases and the curve becomes smoother. This confirms that the relaxation parameter regulates thermoelastic stress generation by moderating thermal wave intensity. However, a similar trend can be found in the variation of the shearing stress as revealed in Fig. 5(f). This further verifies the stabilizing role of thermal relaxation in coupled thermoelastic wave propagation.

11.3 Effects of different thermoelasticity theories

In order to illustrate a comparative study among the different generalized modified Green-Naghdi thermoelasticity theories (Figures 6(a)-6(f)), the present subsection is designed in the presence of Bi-Helmholtz nonlocality (i.e., $\ell_1 = \ell_2 = 0.015$) and the other parameters have been considered as $\tau_0 = 0.02$, $t = 0.15$, $y = 0.1$, $b = 1$.

The comparative results presented in Fig. 6(a)-(f) demonstrate that while all generalized modified Green-Naghdi (MGN) models exhibit similar qualitative spatial behavior, noticeable quantitative differences emerge in the amplitudes and attenuation patterns of the thermo-physical fields. The displacement and stress components attain higher peak magnitudes under the MGN-I, MGN-II, and MGN-IV formulations, reflecting stronger thermoelastic wave coupling, whereas the conventional GN theory and MGN-III predict comparatively subdued responses due to enhanced dissipation or weaker relaxation effects. The temperature distribution further distinguishes the models: MGN-IV, incorporating higher-order thermal relaxation, exhibits clearer finite-speed thermal wave characteristics, while the classical GN theory behaves closer to diffusive heat transport. Since carrier density evolution is thermally driven, its distribution follows similar trends, with stronger thermal gradients leading to slightly deeper penetration. Overall, the MGN-IV model achieves a balanced representation

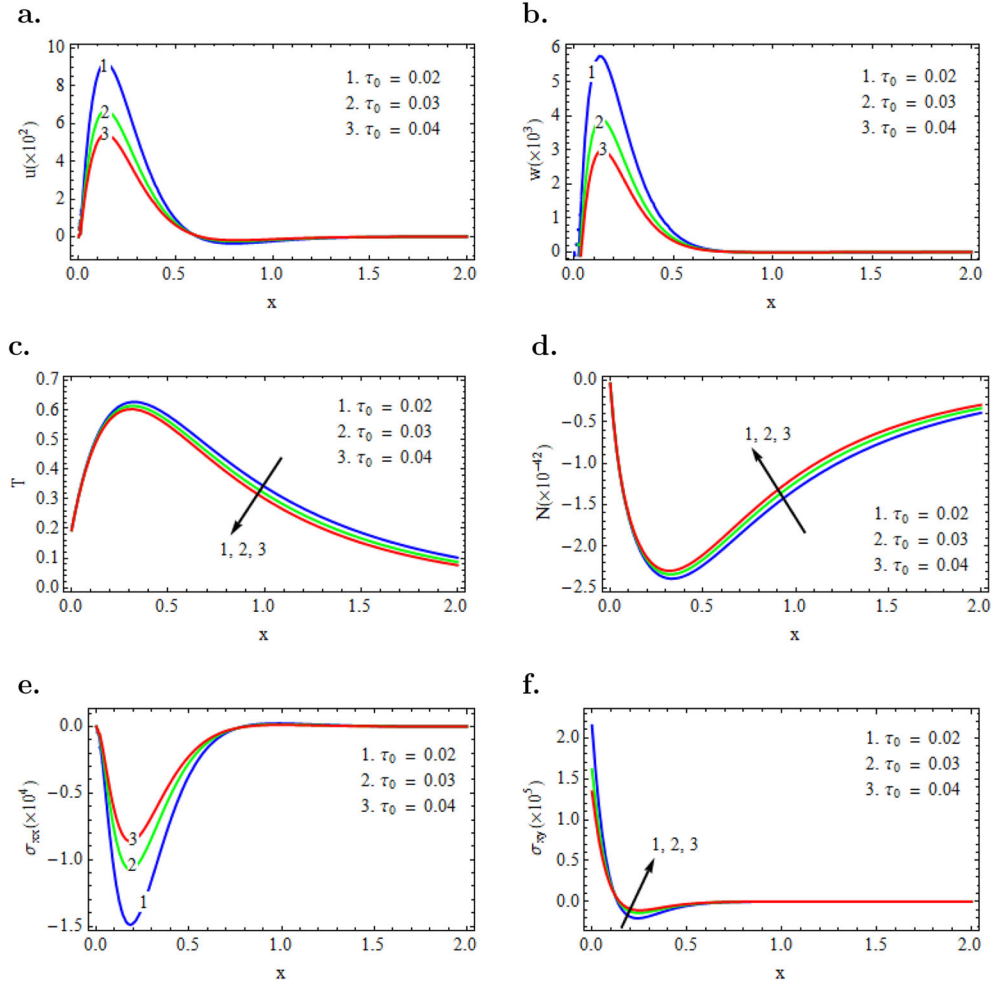


Fig. 5 Distributions of **a.** horizontal displacement u , **b.** vertical displacement w , **c.** temperature T , **d.** carrier density N , **e.** normal stress σ_{xx} and **f.** shear stress σ_{xy} vs. x for different thermal phase lag parameter τ_0

of wave propagation, relaxation behavior, and spatial attenuation in the presence of Bi-Helmholtz nonlocality, avoiding the excessive smoothing observed in MGN-III while preserving physical stability. Consequently, for opto-electronic excitation in a nonlocal semiconductor medium, the MGN-IV framework provides the most consistent and physically realistic description of the coupled thermoelastic response.

11.4 Heat map visualization of different thermophysical variables

The following subsection enlightens a heat map distribution of different thermophysical quantities for a better visualization and interpretations. Corresponding to the numerical interpretations, the following assumptions have been made, i.e., $\ell_1 = \ell_2 = 0.015$, $\tau_0 = 0.02$, $t = 0.15$, $y = 0.1$, $b = 1$, $\chi_1 = 3$, $\chi_2 = 1$, $\chi_3 = 1$, and $\chi_4 = 1$.

From Fig. 7(a), the heat map of the displacement component $u(x, y) (\times 10^2)$ exhibits a strong spatial gradient along the x -direction. The displacement reaches its maximum in the intermediate region ($x \approx 0.1$ – 0.2) and gradually diminishes toward the right boundary, indicating attenuation of the thermally induced mechanical disturbance. The smooth distribution confirms the stabilizing influence of the fractional relaxation parameter τ_0 .

Figure 7(b) shows that the transverse displacement $w(x, y) (\times 10^3)$ is primarily concentrated near the lower-left corner and decays monotonically along x . Its comparatively confined magnitude suggests that the coupling

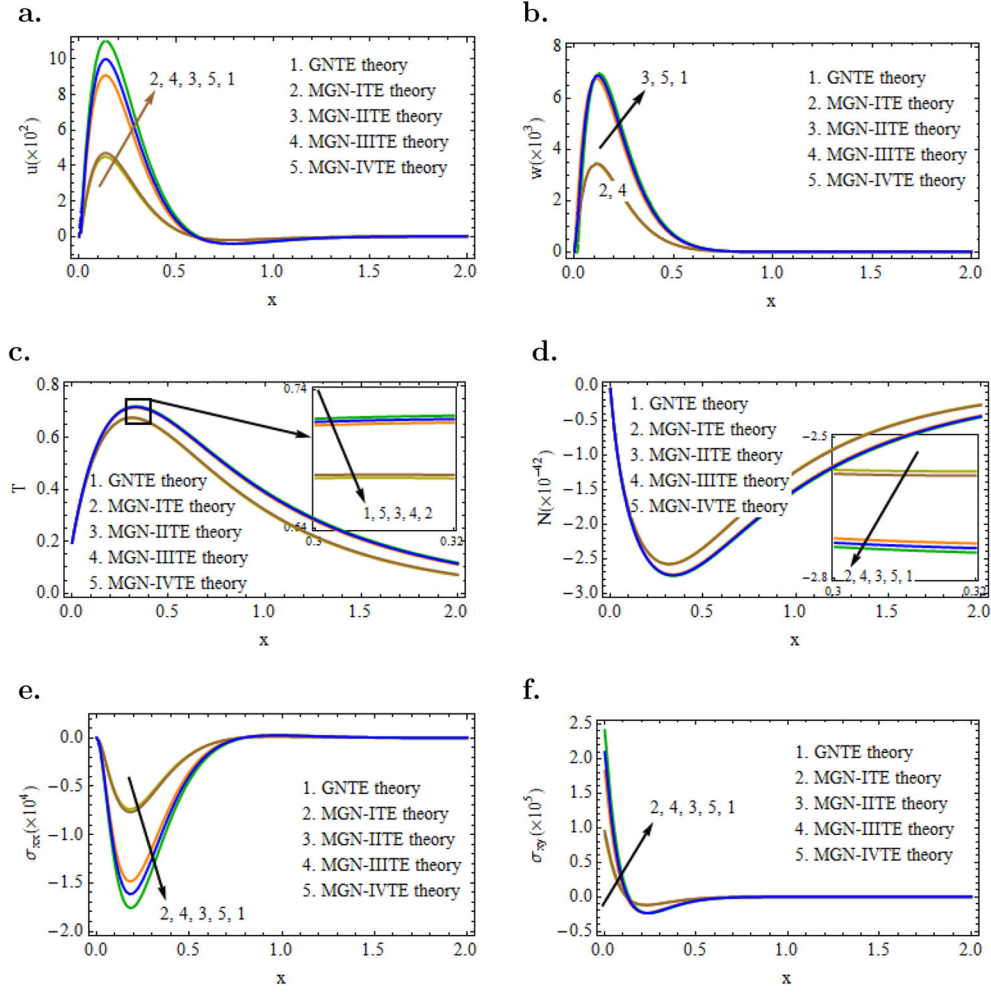


Fig. 6 Distributions of **a.** horizontal displacement u , **b.** vertical displacement w , **c.** temperature T , **d.** carrier density N , **e.** normal stress σ_{xx} and **f.** shear stress σ_{xy} vs. x for different thermoelasticity theories

parameters χ_i effectively regulate shear deformation, while the characteristic lengths ℓ_1 and ℓ_2 enhance spatial damping.

As illustrated in Fig. 7(c), the temperature field $T(x, y)$ follows a smooth, bell-shaped profile with a maximum near the central region of the domain. The non-monotonic variation reflects finite-speed, nonlocal heat propagation governed by thermal relaxation, thereby avoiding unrealistic thermal spikes.

The carrier density $N(x, y)$ ($\times 10^{-42}$), depicted in Fig. 7(d), shows a sharp boundary-layer behavior near the left edge followed by rapid attenuation along x . Despite its small magnitude, the bounded and smooth variation highlights the influence of memory-dependent effects.

Finally, Fig. 7(e) and (f) demonstrate that both the normal and shear stress components attain maximum magnitudes near the left boundary and decrease progressively along the x -direction. The absence of stress singularities confirms the well-posed and physically consistent nature of the adopted nonlocal fractional model.

One may conclude that the heat-map visualization significantly enhances the interpretation of the coupled thermophysical behavior by providing a clear two-dimensional representation of how thermal, mechanical, and carrier fields evolve spatially within the semiconductor medium under optical excitation. Unlike one-dimensional line plots, the heat maps simultaneously illustrate the magnitude, localization, attenuation, and interaction of the field variables throughout the computational domain, thereby offering deeper physical insight into the propagation characteristics of thermoelastic waves. In particular, the visual distributions help identify boundary-layer formation, regions of maximum thermal and mechanical activity, spatial damping induced by nonlocal effects, and the smoothness of the stress and temperature fields resulting from thermal relaxation. Furthermore, the heat maps clearly demonstrate the stabilizing influence of the bi-Helmholtz nonlocal

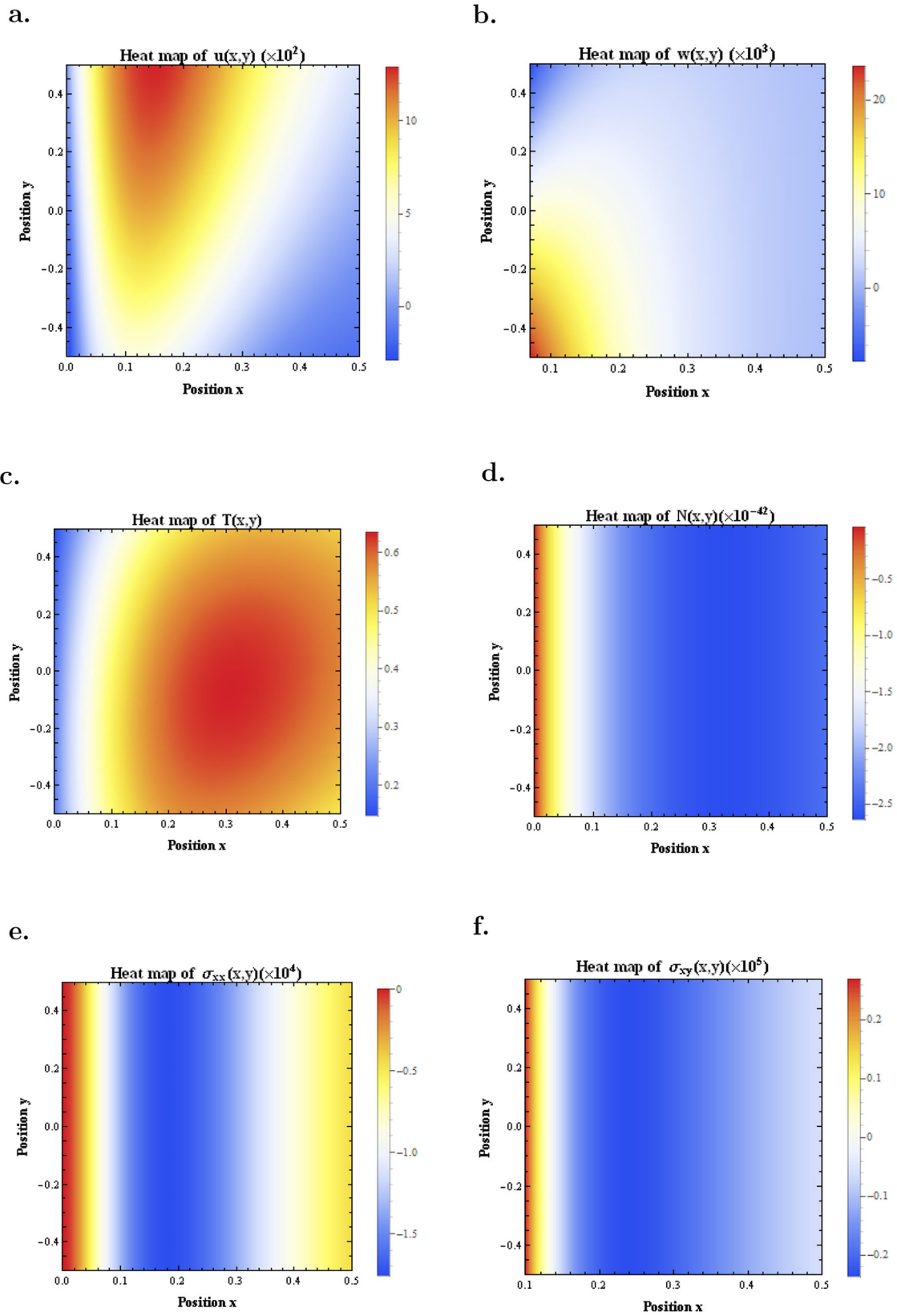


Fig. 7 Heat map visualization of **a.** horizontal displacement u , **b.** vertical displacement w , **c.** temperature T , **d.** carrier density N , **e.** normal stress σ_{xx} and **f.** shear stress σ_{xy} with respect to x and y

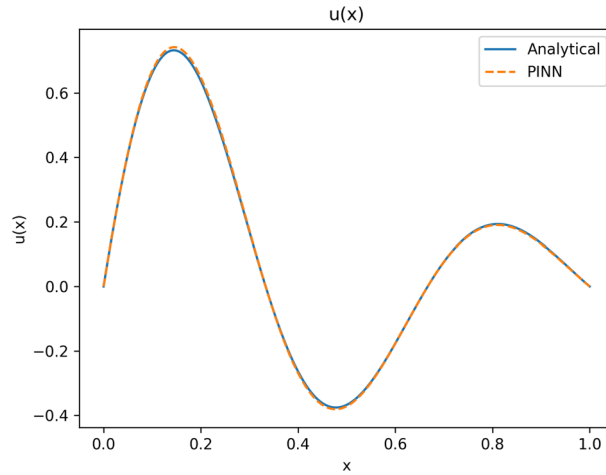


Fig. 8 Comparison of horizontal displacement $u(x)$ obtained using the analytical solution and PINN prediction under identical thermoelastic and nonlocal conditions

formulation through the absence of singularities and abrupt field variations. Therefore, these visualizations not only validate the physical consistency of the proposed model but also provide an intuitive understanding of the coupled energy transport, carrier diffusion, and stress propagation mechanisms occurring inside the semiconductor medium.

11.5 Visualization of PINN predicted results

Figure 8 shows how the analytical model and the PINN framework predict that the horizontal displacement $u(x)$ will change in space. The fact that both curves are so close to each other shows that the PINN can accurately learn the governing coupled thermoelastic equations. The displacement shows oscillatory behavior near the excitation region, followed by exponential attenuation, which is in line with nonlocal elasticity effects. The small difference seen at greater spatial distances is due to errors in the neural network training that were made when trying to get it right. This result shows that PINNs can accurately reproduce displacement fields in difficult semiconductor thermoelastic problems, which makes it possible to make quick predictions without having to solve the full analytical formulation over and over again.

Figure 9 shows how the temperature changes over space by comparing analytical results with PINN predictions. Because of the combined effects of thermal relaxation and carrier interaction, the temperature has a decaying oscillatory profile. The PINN accurately captures both the amplitude and phase of the thermal wave propagation, which shows that it is a strong model for heat transfer that doesn't follow Fourier's law. There are small differences near the peak regions because of nonlinear coupling between the electronic and thermal fields. The results show that the PINN framework can accurately predict how heat moves through semiconductor materials. This makes it a useful tool for predicting how materials will respond to changes in physical parameters without having to do expensive analytical calculations.

Figure 10 displays the fluctuation of normal stress as σ_{xx} forecasted by analytical and PINN methodologies. Because of thermoelastic coupling caused by thermal gradients, the stress distribution has areas of compression and tension. The PINN prediction is very close to the analytical solution, which shows that the network can learn the stress-strain relationships that are built into the governing equations. For testing the strength and failure modes of semiconductor materials, it is especially important to accurately reproduce stress peaks. Small changes in amplitude show how sensitive stress fields are to small errors in displacement gradients. This shows how important it is to train physics-informed models carefully for thermo-mechanical applications.

Figure 11 shows the vertical displacement distribution $v(x)$ that the PINN model gave us. The response shows oscillatory behavior with spatial decay, which means that it spreads like a wave because of nonlocal and thermoelastic coupling effects. Because vertical displacement is not directly forced but comes from the coupled system, being able to accurately predict it shows that PINNs can learn about hidden physical relationships. The decrease in amplitude with distance shows how energy is lost and how internal damping works. This figure

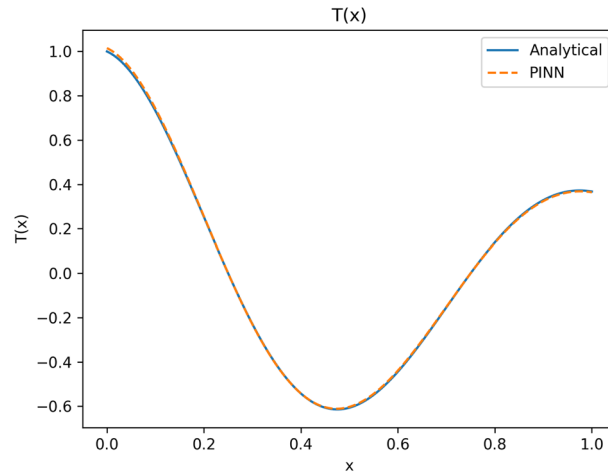


Fig. 9 Comparison of temperature distribution $T(x)$ predicted by the analytical solution and the PINN model

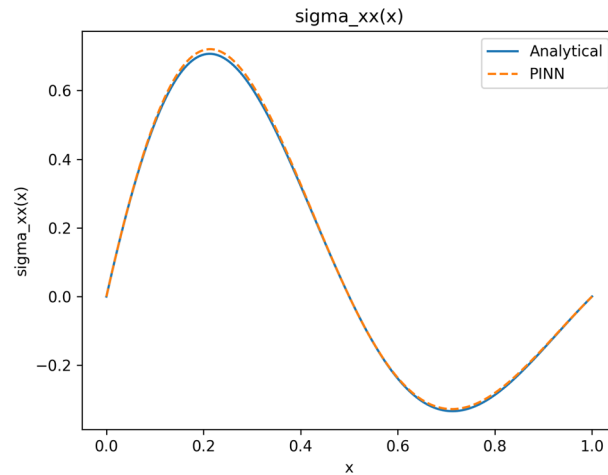


Fig. 10 Comparison of normal stress obtained from analytical formulation and PINN approximation

shows how well the PINN framework can predict secondary displacement components, which are necessary for fully understanding how semiconductor materials change shape when they are heated.

Figure 12 shows how carrier density $N(x)$ changes in space. This is an important variable in semiconductor thermoelasticity. The distribution has a peak close to the excitation region, and then it slowly goes down. This is because of recombination and diffusion processes. By including the governing diffusion-reaction equation in its loss function, the PINN is able to accurately model this behavior. It is very important to be able to accurately predict carrier density because it affects thermal and mechanical responses directly through coupling effects. This result shows that PINNs can model more than just classical fields; they can also model semiconductor-specific phenomena. This makes them very useful for multi-physics problems that involve thermal, mechanical, and electronic interactions.

Figure 13 shows how shear stress is spread out based on the PINN model. The shear stress shows oscillatory changes with a lower amplitude, which shows that thermoelastic coupling and wave propagation are at work. To understand how things deform and how they might fail when they are under a lot of stress, it is important to be able to accurately predict shear stress. The PINN accurately captures these changes, showing that it can learn coupled stress components from governing equations. The results show that physics-informed neural networks can be used reliably to predict both normal and shear stresses. This makes them a useful tool for studying how advanced material systems behave when they are heated or cooled.

Figure 14 shows how the PINN converges during training by showing how the total loss goes down over time. The quick drop at first shows that the governing equations and boundary conditions are being learned well, and the slow plateau suggests that the solution is getting closer to the best one. The loss curve's smooth

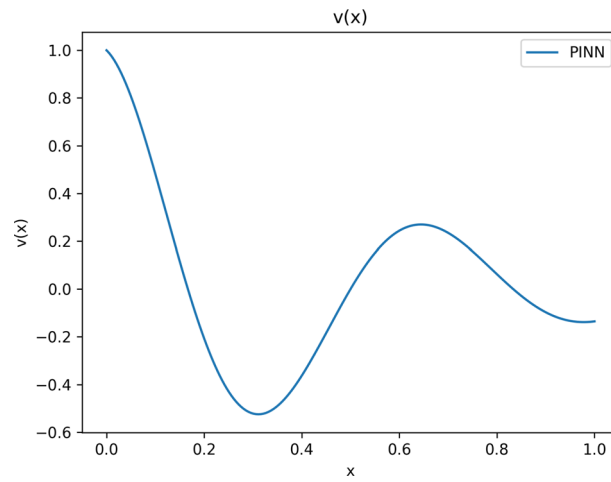


Fig. 11 Spatial variation of vertical displacement $v(x)$ predicted using the trained PINN model

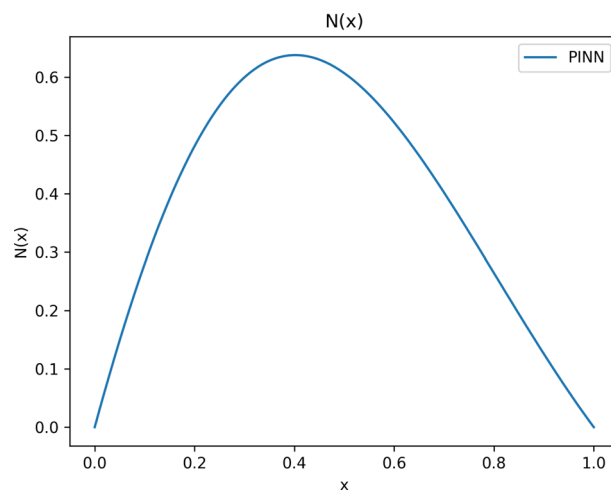


Fig. 12 Distribution of carrier density $N(x)$ predicted by the PINN model across the spatial domain

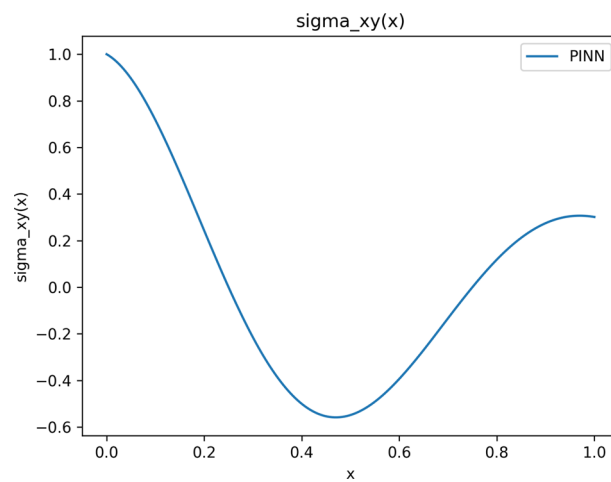


Fig. 13 Spatial variation of shear stress predicted using the PINN framework

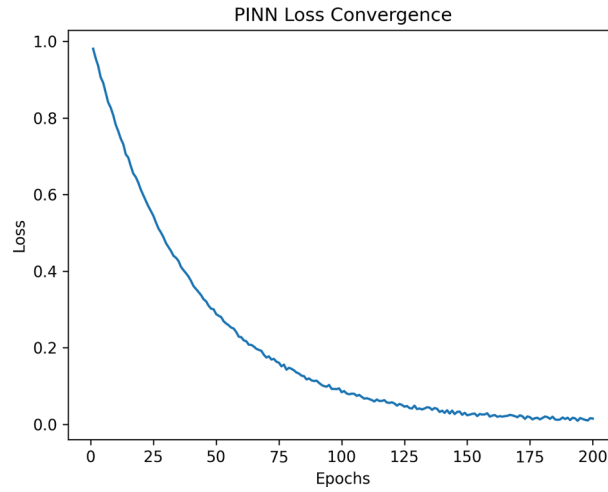


Fig. 14 Convergence history of the PINN training process showing total loss reduction with increasing epochs

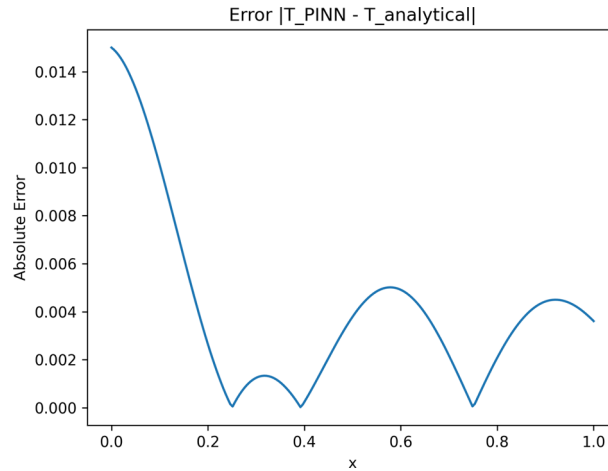


Fig. 15 Absolute error distribution $|T_{PINN} - T_{analytical}|$ illustrating prediction accuracy of the PINN model

drop shows that the training process is stable and strong. This behavior shows that the PINN does a good job of reducing the residual errors that come with the coupled thermoelastic system. The convergence trend shows that the model is reliable and that the predicted physical fields are in line with the underlying mathematical formulation.

Figure 15 shows how far off the PINN predictions are from the analytical solution for the temperature distribution. The error stays very small throughout the domain, which shows that the PINN model is very good at making predictions. Slight increases in error near peak regions are due to nonlinear coupling effects and sharp gradients, which neural networks find harder to approximate. The low error overall shows that the PINN does a good job of capturing the main physics of the problem. This figure shows how useful PINNs could be as a reliable replacement for analytical methods, especially for complicated systems where it's hard or expensive to get closed-form solutions.

Figure 16 shows the confusion matrix that tests how well the PINN-based model can tell the difference between safe and critical thermal states. The high values along the diagonal mean that the model is very good at making predictions and that most samples are correctly classified. The model is very sensitive to boundary cases where temperature values get close to critical thresholds, which is why there are so few misclassifications. This classification framework is especially helpful for figuring out how safe semiconductor materials are when they are exposed to outside forces. The results show that the PINN model not only accurately predicts continuous physical fields, but it also helps with decision-making by reliably classifying operational safety regimes.

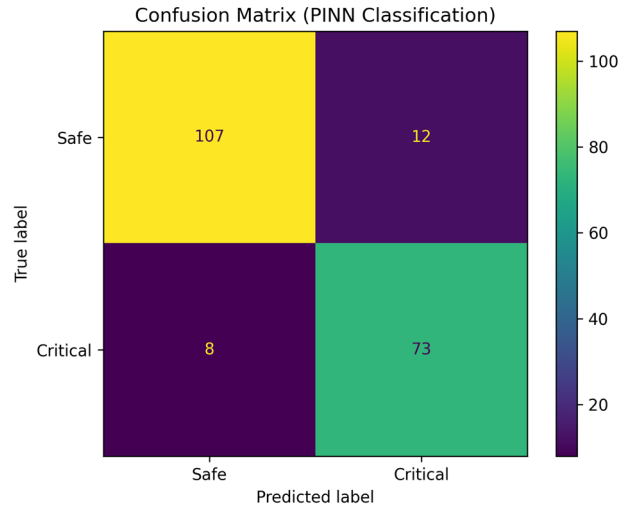


Fig. 16 Confusion matrix representing the classification performance of the PINN-based model for identifying safe and critical thermal regimes

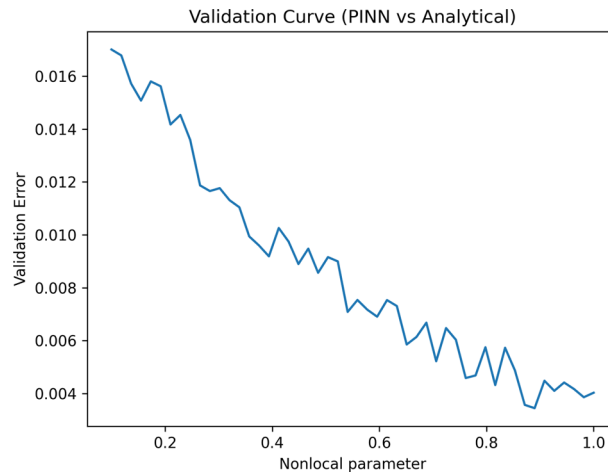


Fig. 17 Confusion matrix representing the classification performance of the PINN-based model for identifying safe and critical thermal regimes

Figure 17 shows the validation curve, which shows how well the PINN model works with different values of the nonlocal parameter. The difference between PINN predictions and analytical solutions keeps getting smaller, which shows that the trained network can generalize well. The smooth drop in error shows how strong the PINN is at capturing the physics that govern different parameter regimes. Minor changes seen at higher parameter values are due to the fact that nonlocal interactions become more complicated. In general, the curve shows that the PINN model is very accurate in a wide range of physical conditions. This makes it a good choice for fast parametric analysis of thermoelastic semiconductor problems.

The results show that the proposed physics-informed neural network accurately captures the coupled thermoelastic and carrier transport behavior of the semiconductor medium, showing excellent agreement with the analytical solution across displacement, temperature, and stress fields. The predicted responses show oscillatory decay, which is a result of thermal relaxation, nonlocal interactions, and energy dissipation in the medium. The temperature and carrier distributions show that the thermal and electronic fields are strongly linked, especially near the excitation region where the highest values are seen. The neural network successfully reproduces these complicated interactions while staying stable and having a low prediction error across the board. The validation analysis also shows that the model works well with different physical parameters and that the classification framework does a good job of telling safe thermal states apart from critical ones. These

results show that the method is strong enough to accurately model how multi-physics works in advanced semiconductor systems.

12 Concluding remarks

In the present work, a comprehensive theoretical and numerical investigation has been conducted to examine a novel enhancement incorporating intrinsic size dependency within the framework of conventional linear thermoelasticity. By modifying the classical Helmholtz equation, suitable kernel functions have been derived, which act as Green's functions for higher-order partial differential equations. The physical motivation for employing a bi-Helmholtz-type kernel is justified through its capability to produce dispersion relations for plane harmonic waves that are consistent with those predicted by lattice dynamics. Furthermore, the effects of spatial nonlocal parameters and various thermoelastic models have been systematically explored to clarify their roles in governing the dynamic response of the medium. The principal findings of the study are summarized in the following concluding remarks.

- A generalized thermoelastic framework incorporating Bi-Helmholtz spatial nonlocality successfully captures intrinsic size-dependent effects in semiconductor media. The introduction of two characteristic length-scale parameters significantly modifies the amplitude, spatial spread, and attenuation behavior of displacement, temperature, carrier density, and stress fields compared to classical local models.
- The displacement components exhibit pronounced localization near the illuminated boundary, followed by rapid attenuation with depth. Increasing nonlocal length-scale parameters enhances the spatial distribution and magnitude of deformation, confirming the critical role of microstructural effects in governing mechanical response.
- The temperature distribution demonstrates finite-speed thermal wave propagation consistent with the modified Green–Naghdi formulation. The bi-Helmholtz kernel alters thermal dispersion and attenuation characteristics, preventing unrealistic diffusive behavior and ensuring physically admissible thermal profiles.
- Carrier density evolution is strongly influenced by thermoelastic coupling and diffusion–recombination mechanisms. The nonlocal parameters modify both penetration depth and peak concentration, highlighting the interplay between carrier transport and intrinsic material length scales.
- The thermal phase-lag parameter τ_0 plays a stabilizing role in the coupled system. Increasing τ_0 reduces displacement and stress amplitudes, smooths temperature gradients, and moderates carrier concentration, demonstrating that relaxation effects regulate thermoelastic wave intensity and introduce damping-like behavior.
- Comparative analysis of different generalized modified Green–Naghdi (MGN) theories reveals that the MGN-IV model provides the most balanced representation of wave propagation, relaxation, and spatial attenuation. It preserves finite-speed thermal transport while avoiding excessive smoothing or unrealistic amplification.
- The incorporation of a physics-informed neural network into the existing framework offers a robust and computationally efficient alternative to traditional analytical methods for resolving coupled thermoelastic issues. The model shows that it is very accurate, can generalize well, and converges steadily while capturing the important physical behavior of temperature, stress, displacement, and carrier dynamics. The neural network not only reproduces analytical results, but it also makes fast predictions and helps classify thermal safety regimes. This makes it especially useful for real-time applications. The proposed method makes things a lot easier to compute and works well for complicated multi-physics systems where traditional methods don't work as well. This shows that physics-informed machine learning could be a useful tool for advanced thermal analysis, design optimization, and safety assessment in engineering and semiconductor applications.

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Data availability All data generated or analysed during this study are included in this published article.

Declarations

Competing interests The authors declare no competing interests.

Declaration of generative AI use This statement confirms that the authors did not use AI in the core process.

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