RECEP TAYYİP ERDOĞAN ÜNİVERSİTESİ
Mühendislik ve Mimarlık Fakültesi
İnşaat Mühendisliği Bölümü

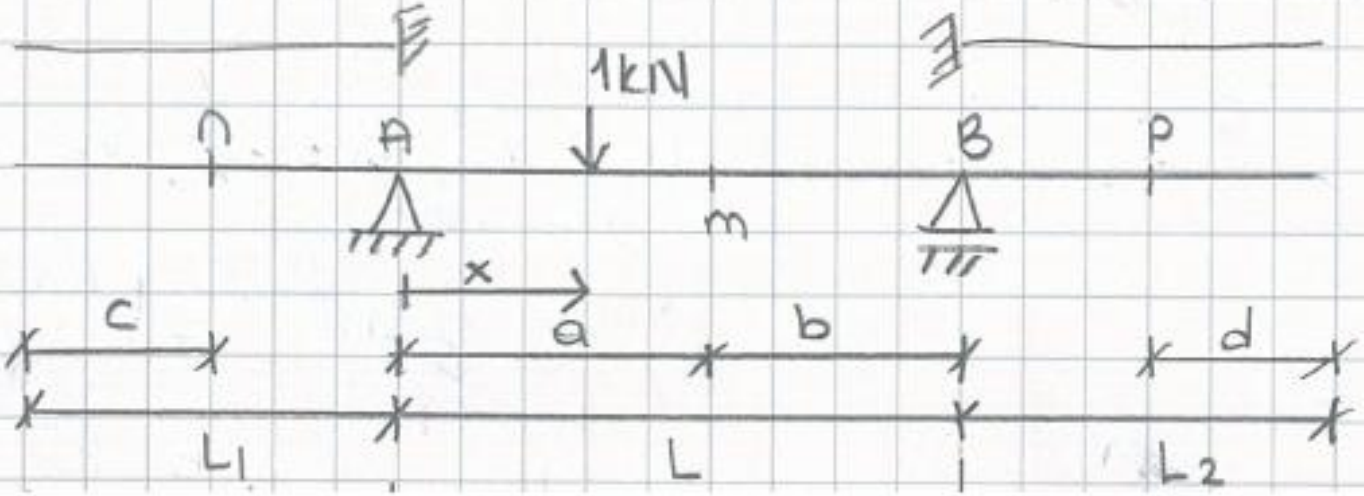
INS313 YAPI STATİĞİ-I

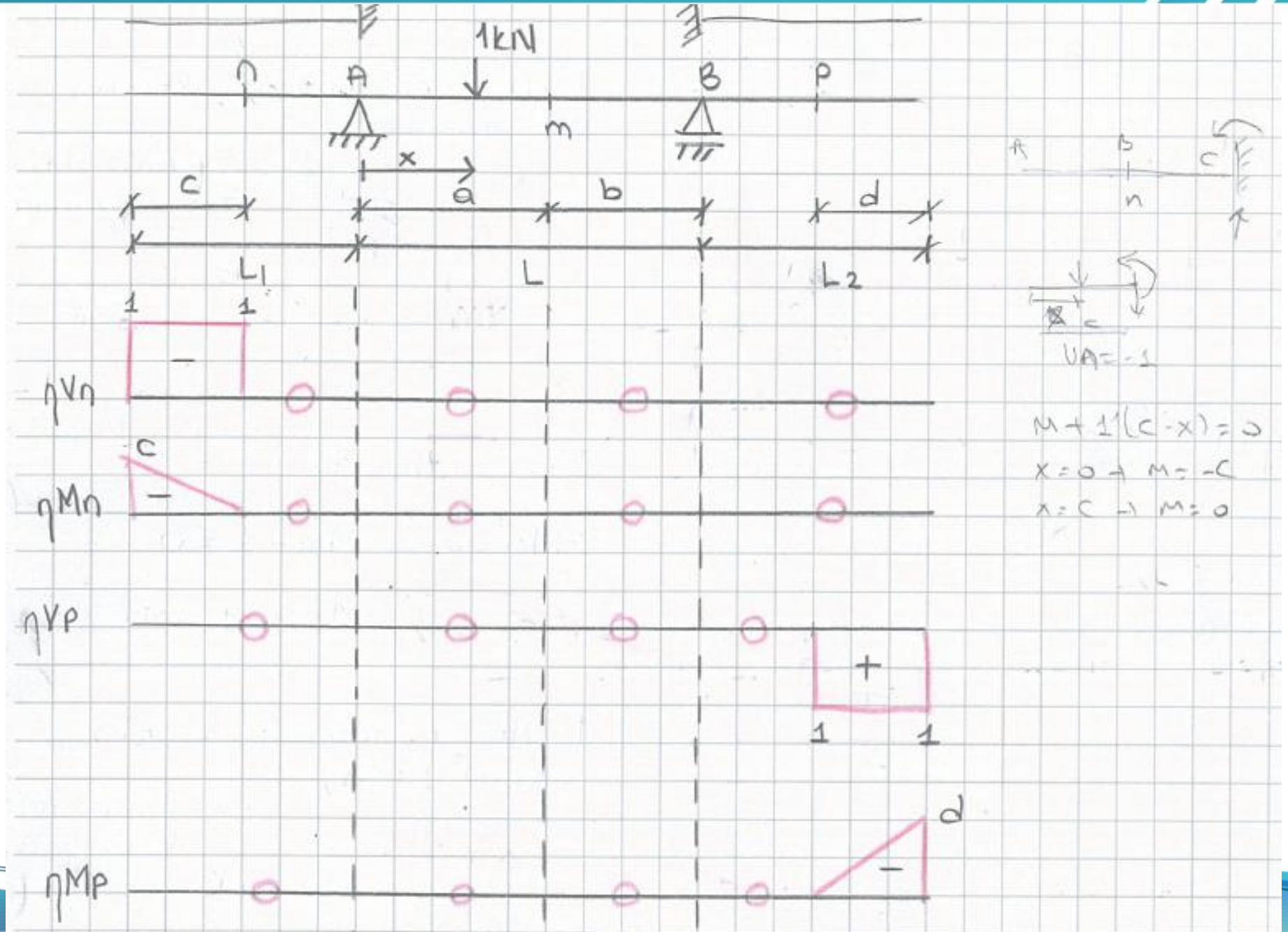
Dr. Öğr. Üyesi Zeliha TONYALI

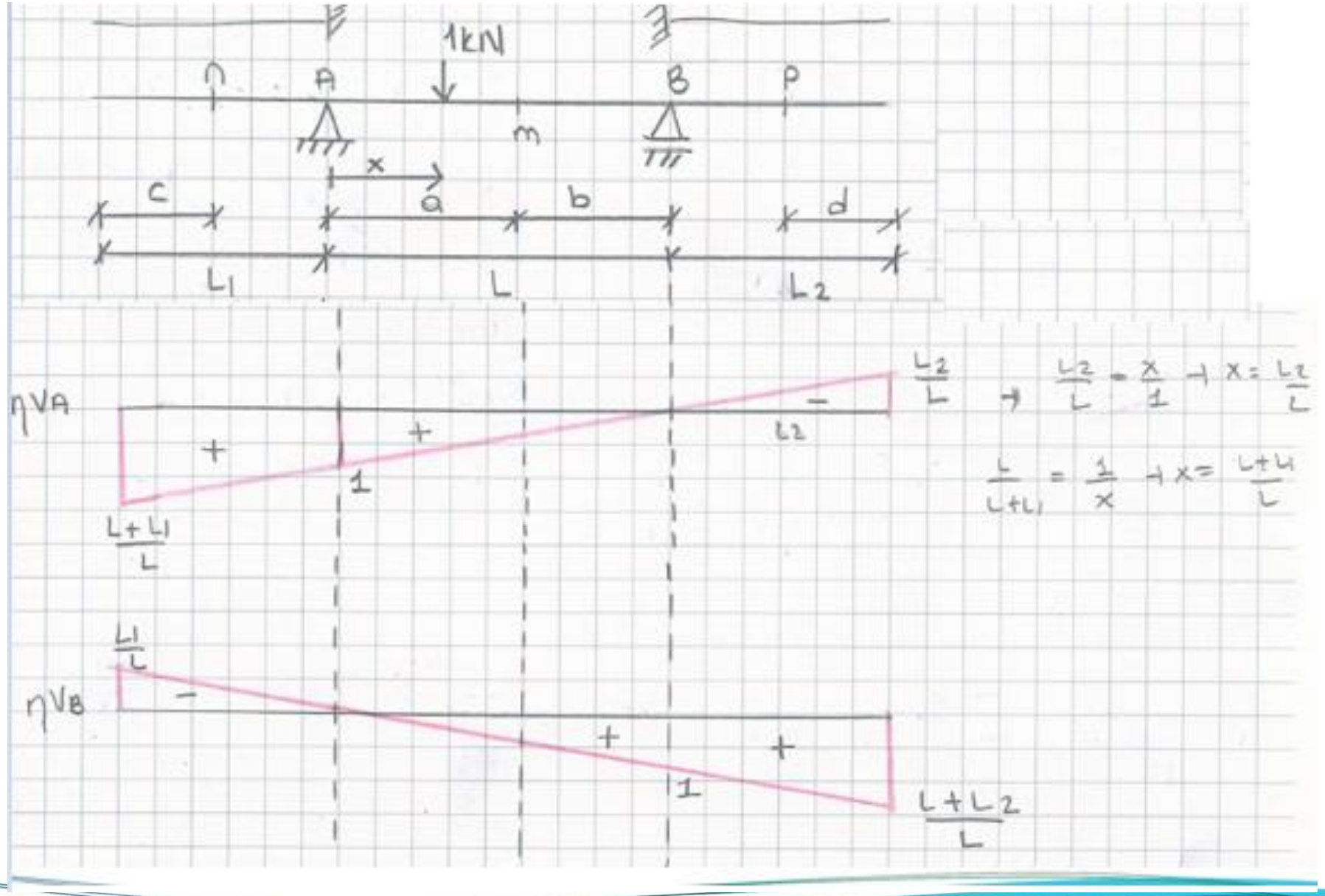
‘’13H. ÇIKMALI KİRİŞLERDE TESİR ÇİZGİLERİ DİYAGRAMLARI”

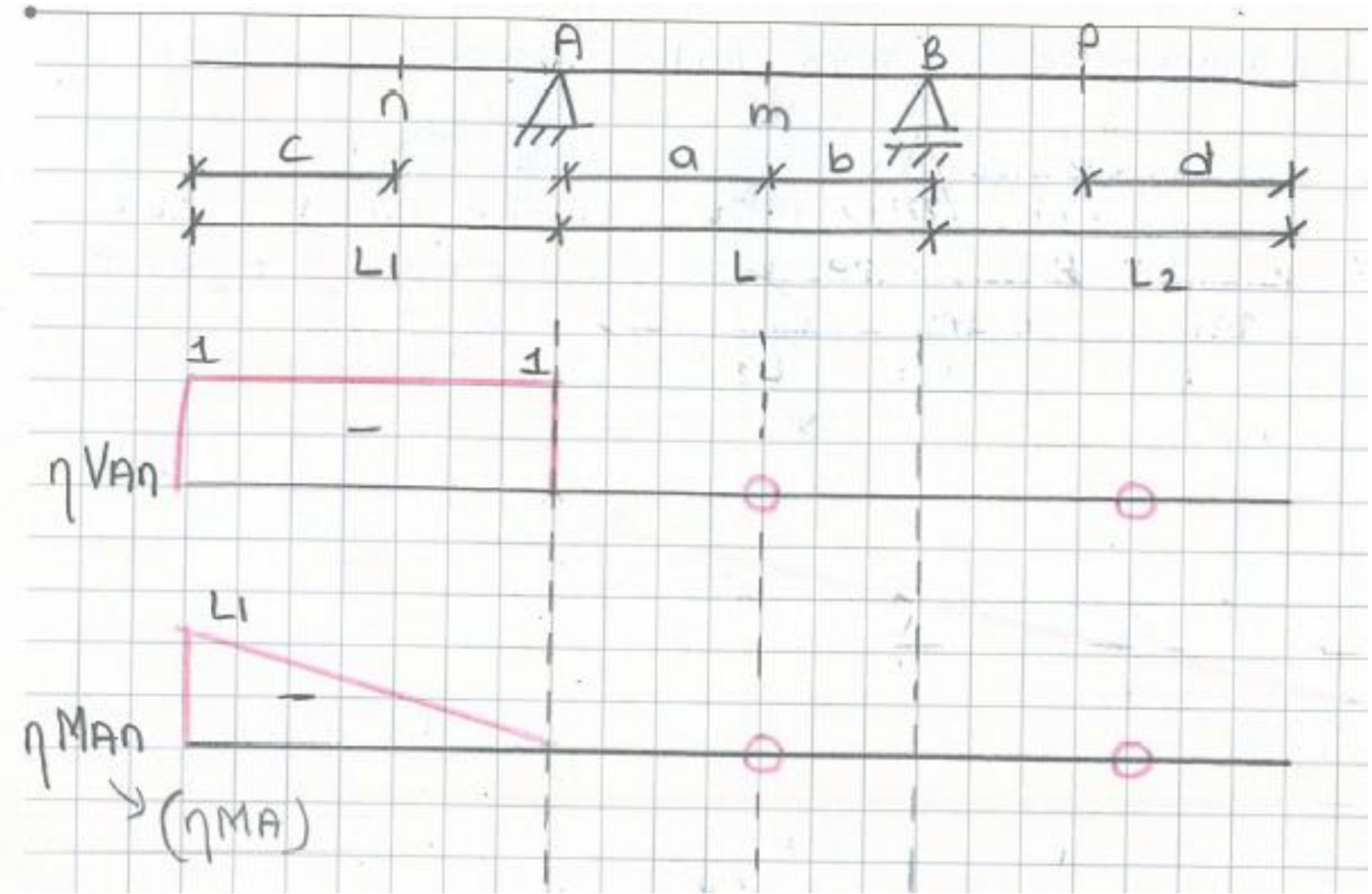
Çıkmalı Kirişlerde Tesir Çizgileri

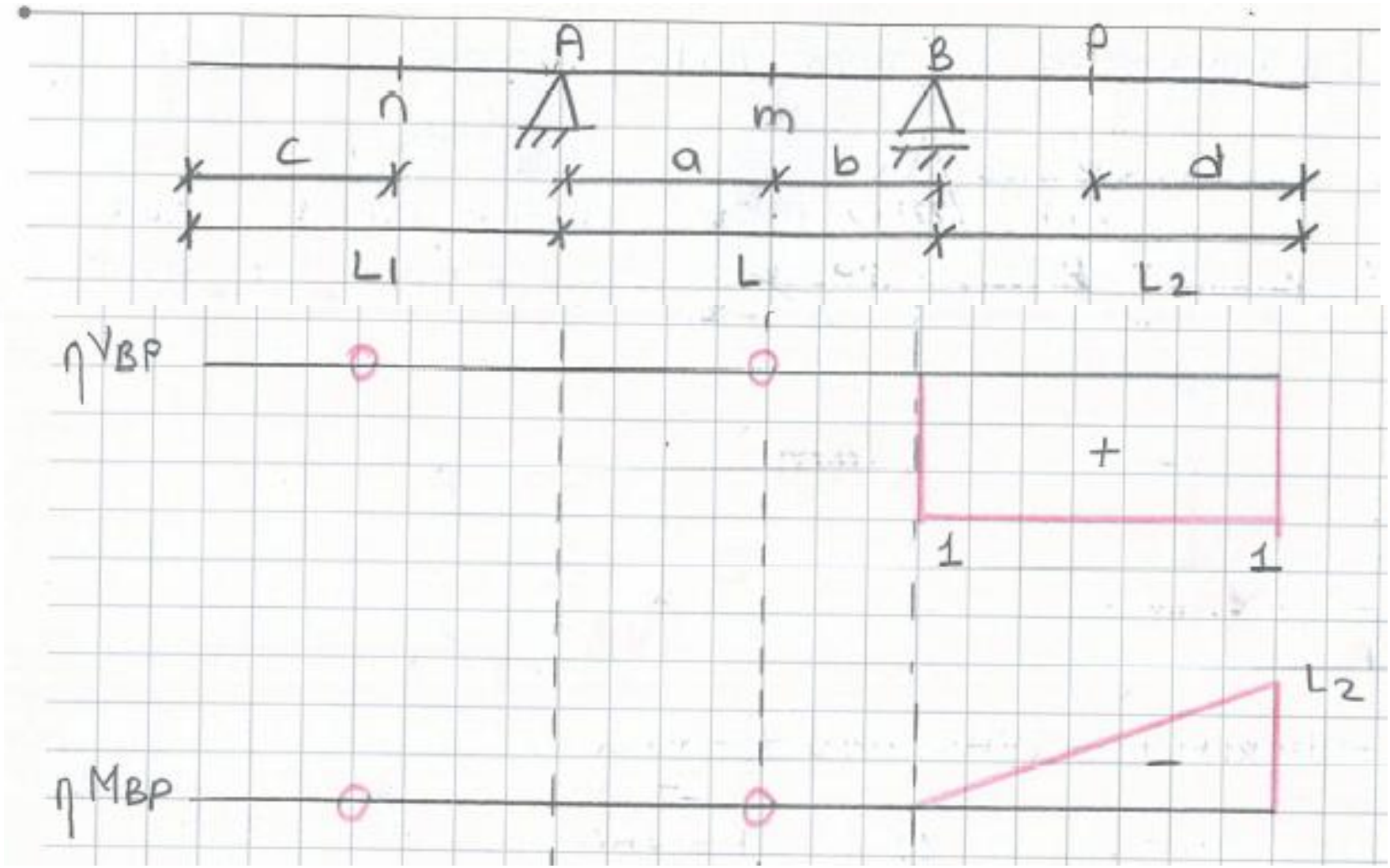
İki ucu çıkmalı bir kiriş üzerine 1kN'luk düşey bir kuvvet hareket ederken çıkmalar üzerindeki kesitlere ait tesir çizgileri konsol kirişlerin aynısıdır. Çıkmalı kirişin mesnetler arasındaki bir m kesitine ait tesir çizgilerini tayin etmek için önce basit kirişin m kesitine ait tesir çizgileri çizilir. Daha sonra bu doğru parçaları konsol kısımlarda uçlara kadar uzatılır.

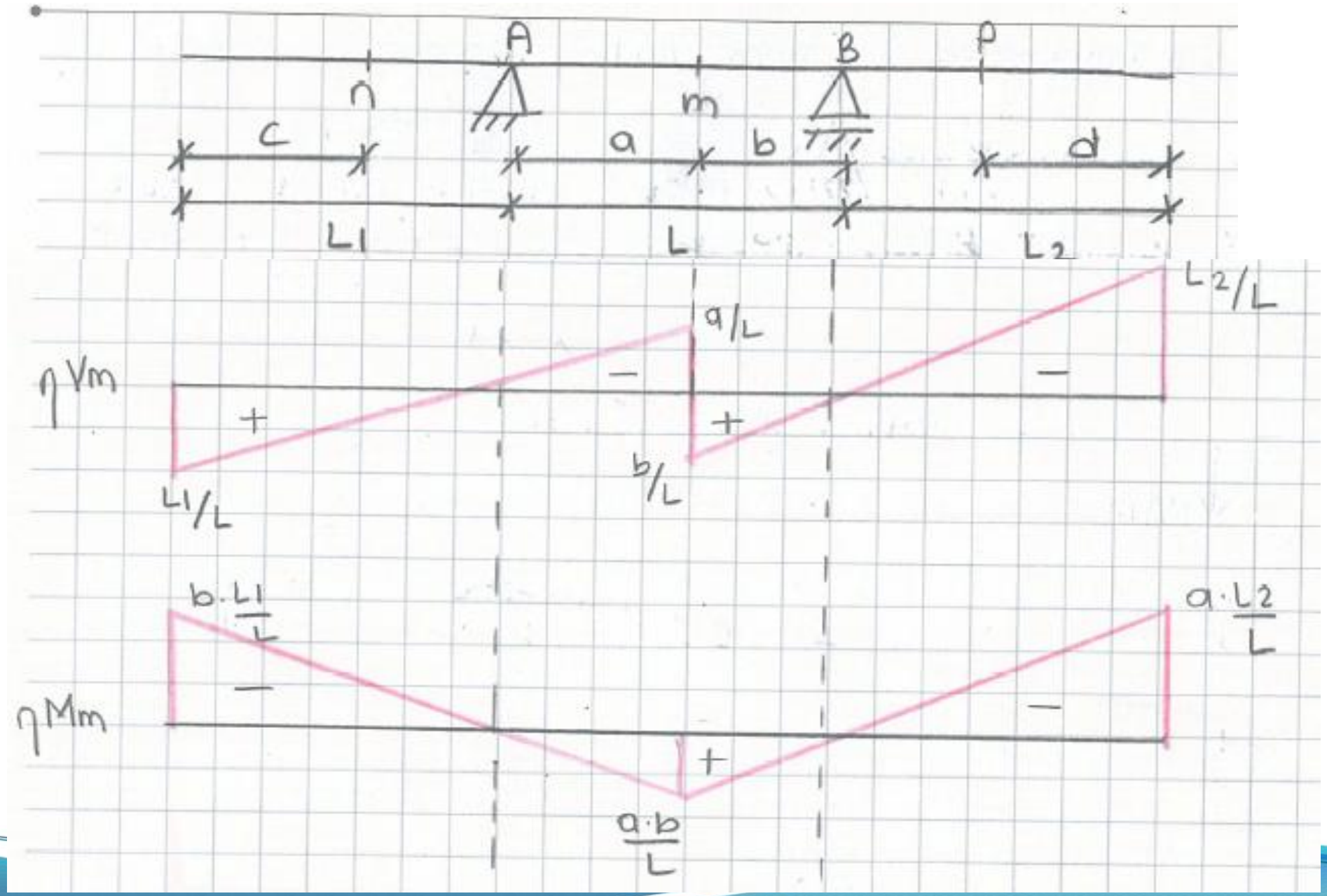


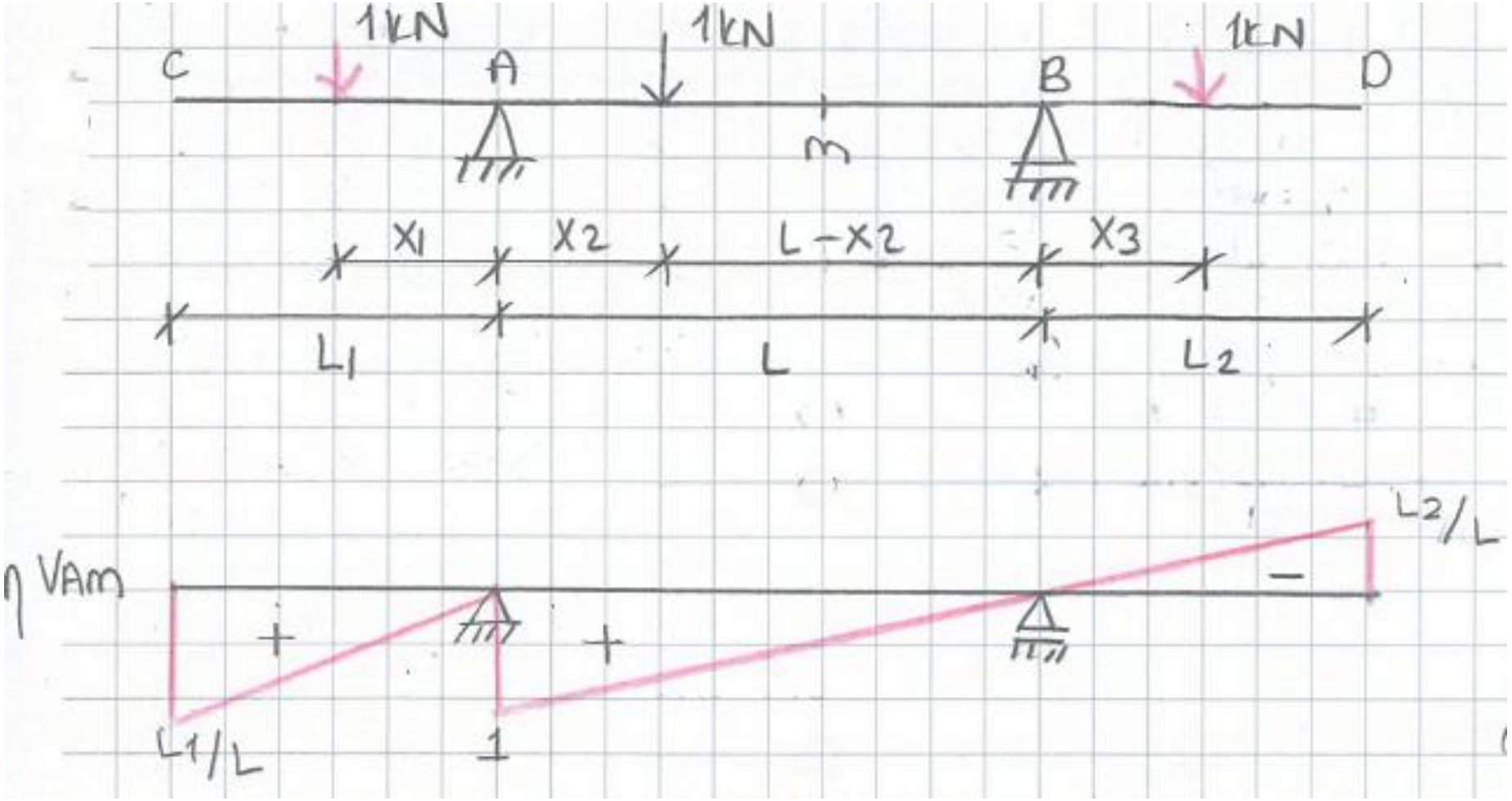

1kN
n
A
B
p
m
x
c
a
b
d
L1
L
L2
1
1
ηVn
c
ηMn
ηVp
+
1
1
d
ηMp
A
B
C
n
UA = -1
M + 1(c - x) = 0
x = 0 → M = -c
x = c → M = 0


1kN
A
B
m
x
c
a
b
d
L1
L
L2
ηVA
ηVB
L2/L
L+L1/L
L1/L
L+L2/L
1
1
+
+
+
+
+
−
−
L2/L → L2/L = x/1 → x = L2/L
L/L+L1 = 1/x → x = L+L1/L


A
B
P
n
m
c
a
b
d
L1
L
L2
1
1
ηVAn
L1
ηMAn
(ηMA)


A
B
P
n
m
c
a
b
d
L1
L
L2
ηVBP
+
1
1
L2
ηMBP
−

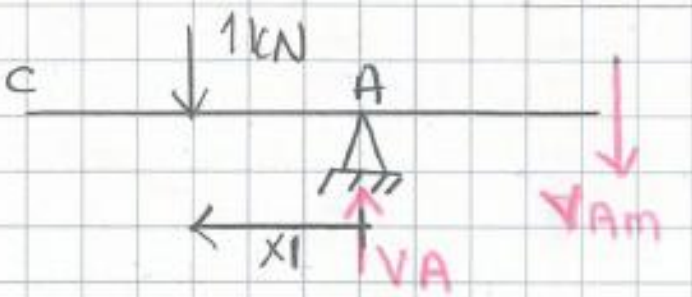

A
B
P
n
m
c
a
b
d
L1
L
L2
L2/L
a/L
ηVm
+
−
−
+
b/L
L1/L
b·L1/L
a·L2/L
ηMm
−
−
+
a·b/L

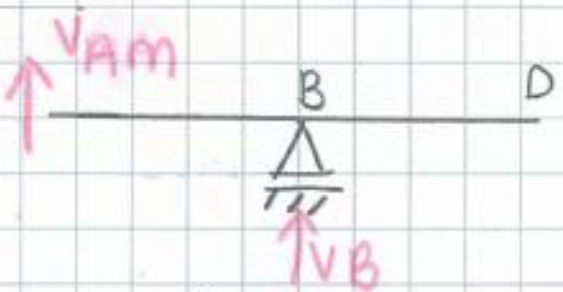

1kN
1kN
1kN
C
A
B
D
m
X1
X2
L-X2
X3
L1
L
L2
η VAm
L2/L
L1/L
1
+
+
−



Kontrol

1 kN luk yük CA arasında $0 \le x_1 \le L_1$



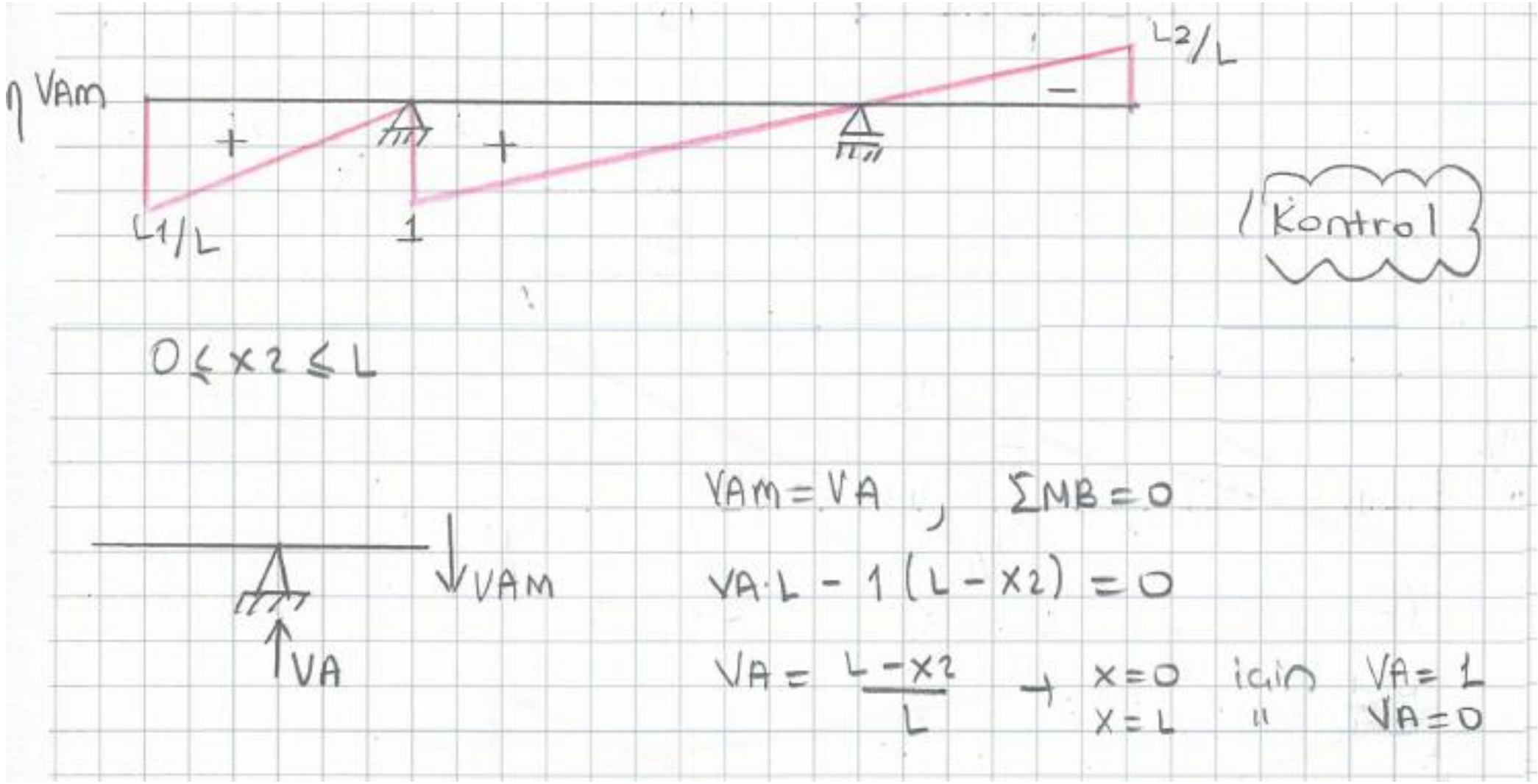


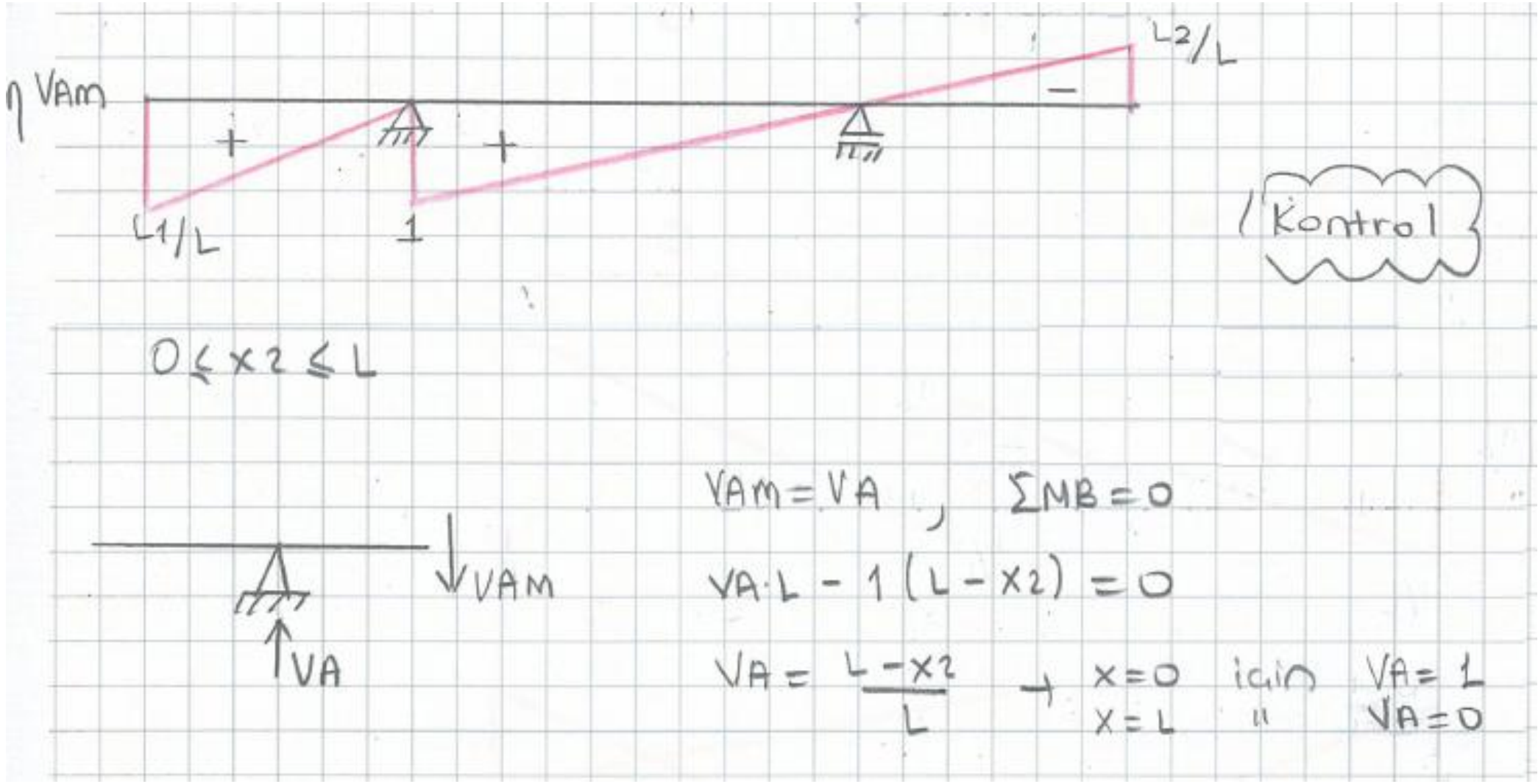
$\Sigma M_B = 0 \quad V_A \cdot L = 1 \cdot (x_1 + L) \rightarrow V_A = \frac{x_1}{L} + 1$ (Tüm sistemden)

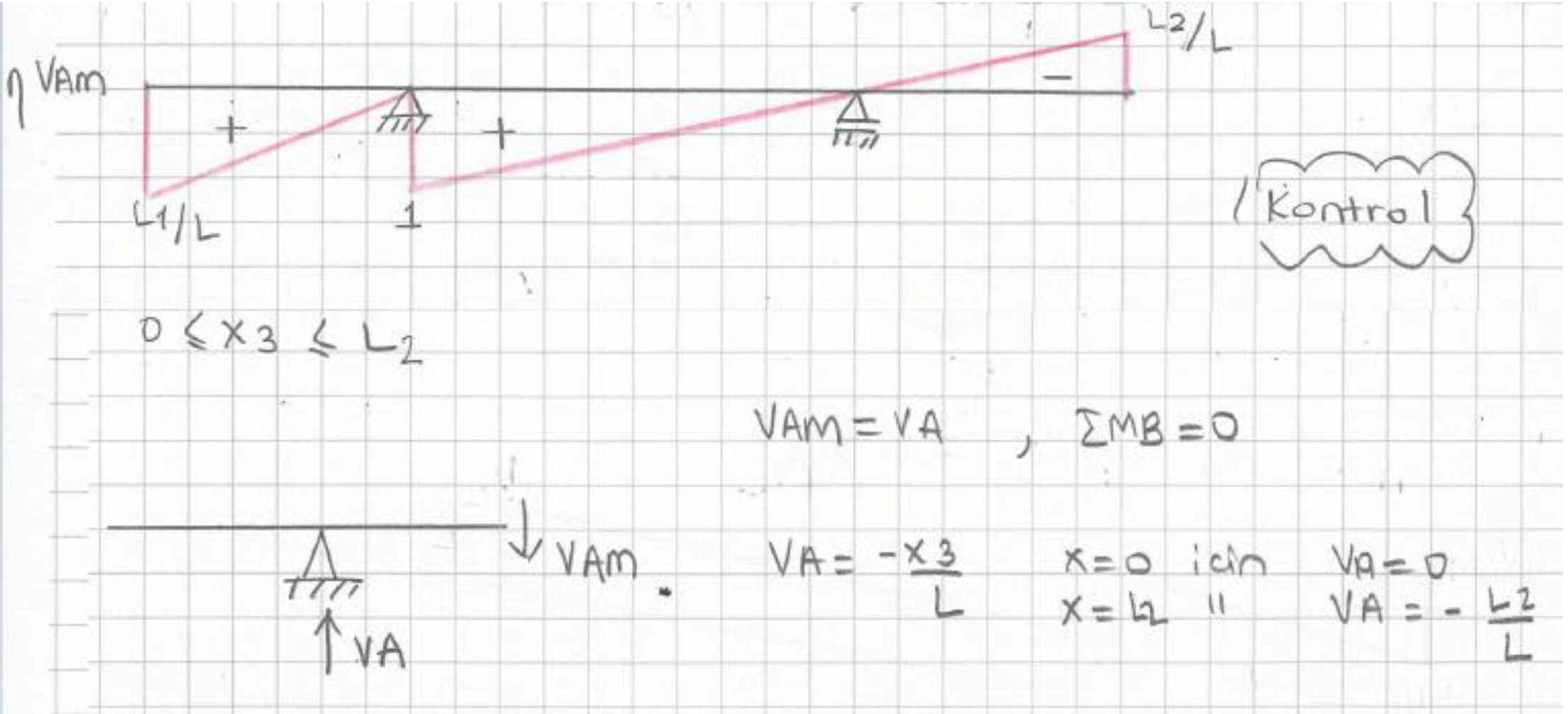
$\uparrow \Sigma Y = 0 \quad V_A - 1 - V_{AM} = 0 \rightarrow \frac{x_1}{L} + 1 - 1 = V_{AM} \rightarrow V_{AM} = \frac{x_1}{L}$

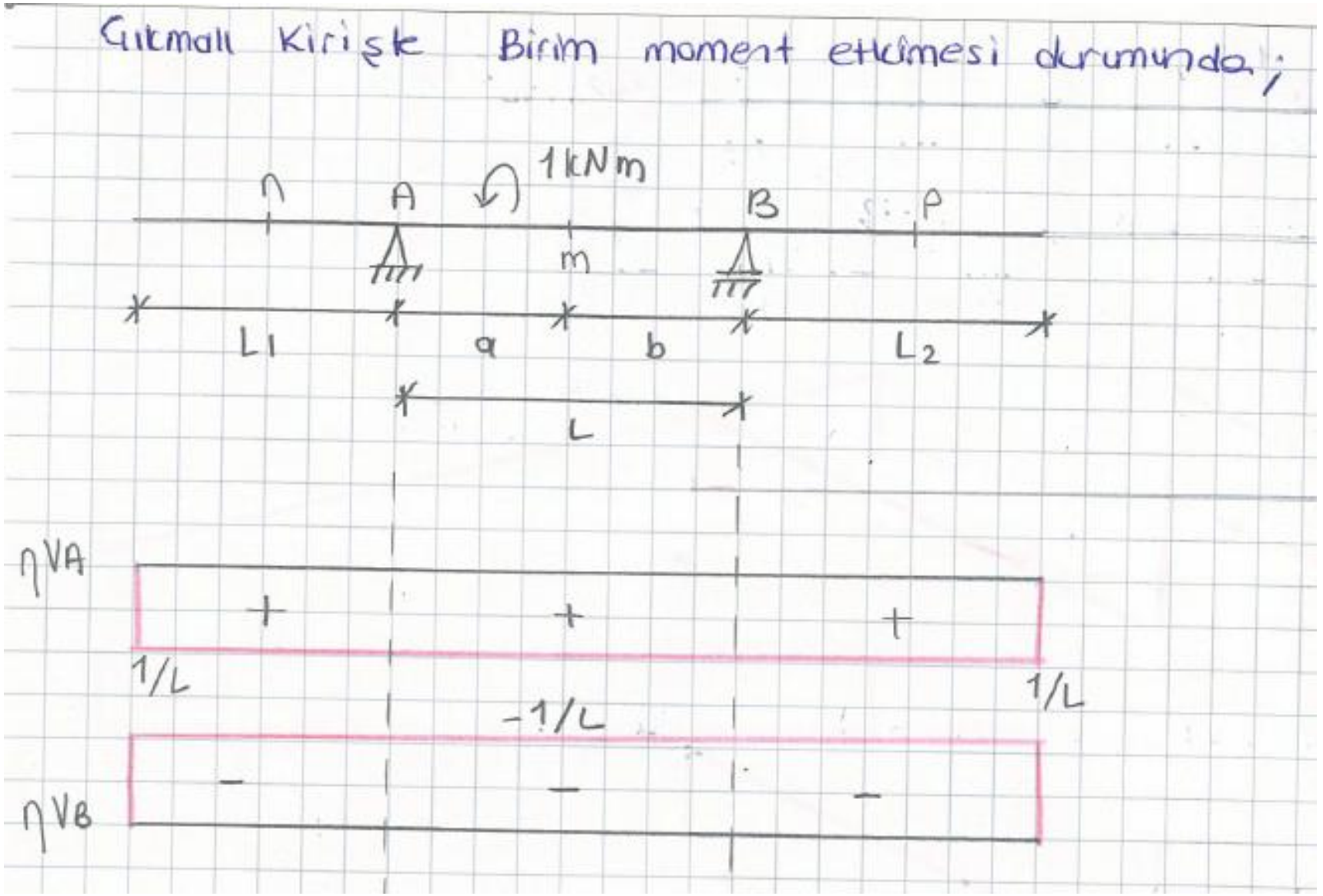
$x_1 = 0 \rightarrow V_{AM} = 0$

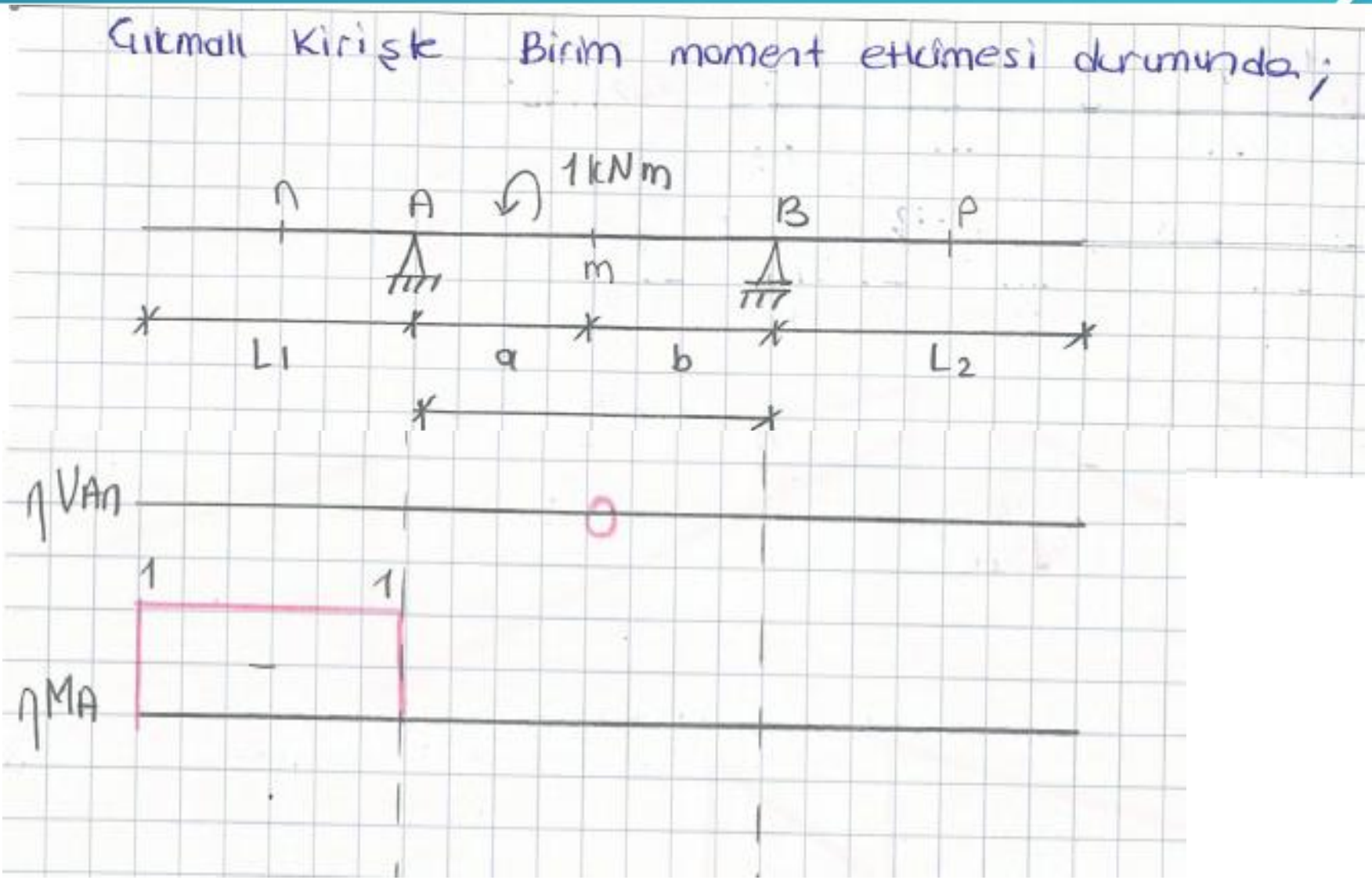
$x_1 = L_1 \rightarrow V_{AM} = \frac{L_1}{L}$


η VAM
+
+
−
L1/L
1
L2/L
Kontrol
0 ≤ x2 ≤ L
VAM
VA
VAM = VA , ΣMB = 0
VA·L − 1(L − x2) = 0
VA = (L − x2)/L → x=0 için VA=1
x=L " VA=0

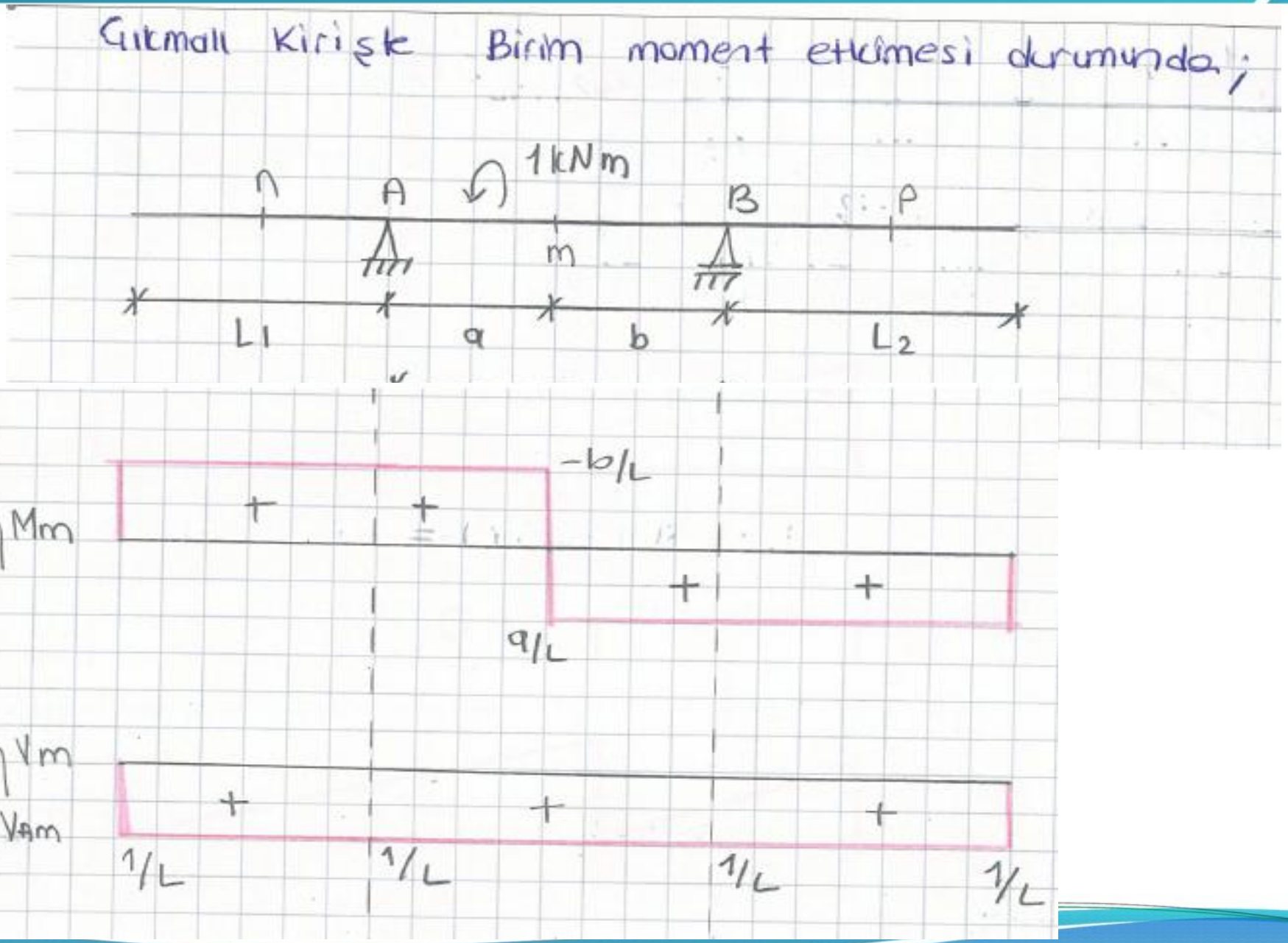

η VAM
+
+
−
L1/L
1
L2/L
Kontrol
0 ≤ x2 ≤ L
VAM
VA
VAM = VA , ΣMB = 0
VA·L − 1(L − x2) = 0
VA = (L − x2)/L → x=0 için VA=1
x=L " VA=0


η VAm
+
+
−
L1/L
1
L2/L
Kontrol
0 ≤ x3 ≤ L2
VAm = VA , ΣMB = 0
VAm
VA
VA = -x3/L
x = 0 için VA = 0
x = L2 " VA = - L2/L

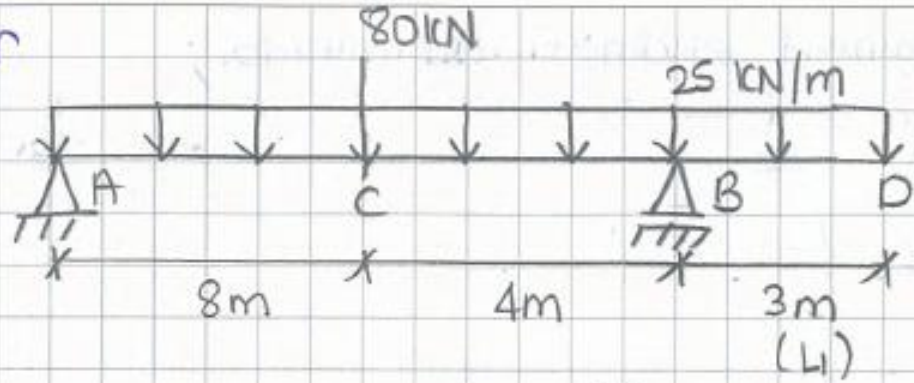

Çıkmalı Kirişte Birim moment etkimesi durumunda;
1kNm
A
B
P
m
L1
a
b
L2
L
ηVA
+
+
+
1/L
1/L
-1/L
ηVB
-
-
-


Gitmali Kirişk Birim moment etkimesi durumunda;
1kNm
A
B
P
m
L1
a
b
L2
VAn
MA
1
1
−

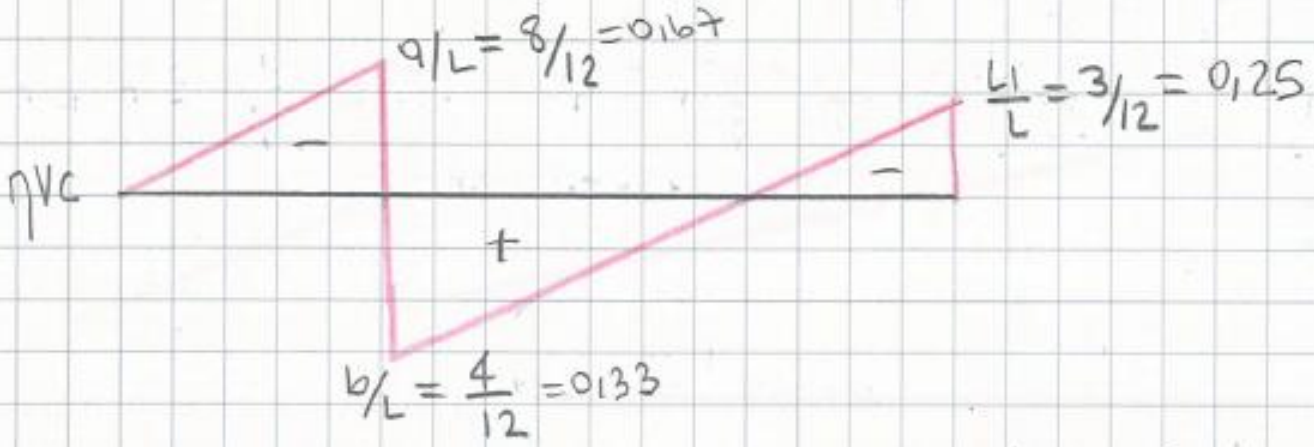
Gitmali Kirişk Birim moment etkimesi durumunda;



Ör 7

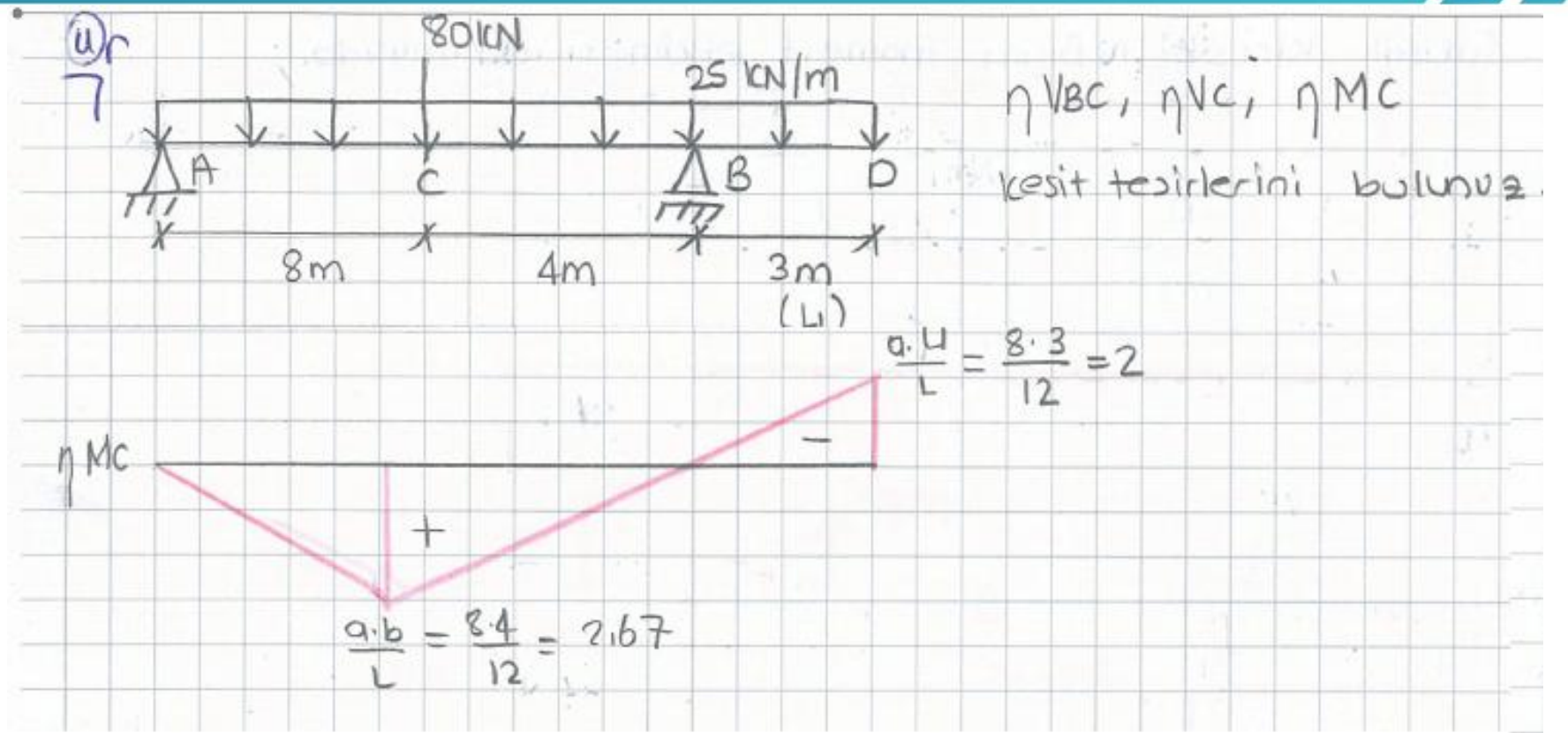


ηV_{BC}, ηV_C, ηM_C kesit tesirlerini bulunuz.



$$V_{CA} = 25\cdot\left(-\frac{8}{12}\cdot 8\cdot\frac{1}{2} + \frac{4}{12}\cdot 4\cdot\frac{1}{2} - \frac{3}{12}\cdot 3\cdot\frac{1}{2}\right) + 80\left(\frac{4}{12}\right) \Rightarrow V_{CA} = -32{,}76\ kN$$

$$V_{CB} = 25\left(-\frac{8}{12}\cdot 8\cdot\frac{1}{2} + \frac{4}{12}\cdot 4\,\frac{1}{2} - \frac{3}{12}\cdot 3\cdot\frac{1}{2}\right) + 80\cdot\left(-\frac{8}{12}\right) \Rightarrow V_{CB} = -112{,}71\ kN$$



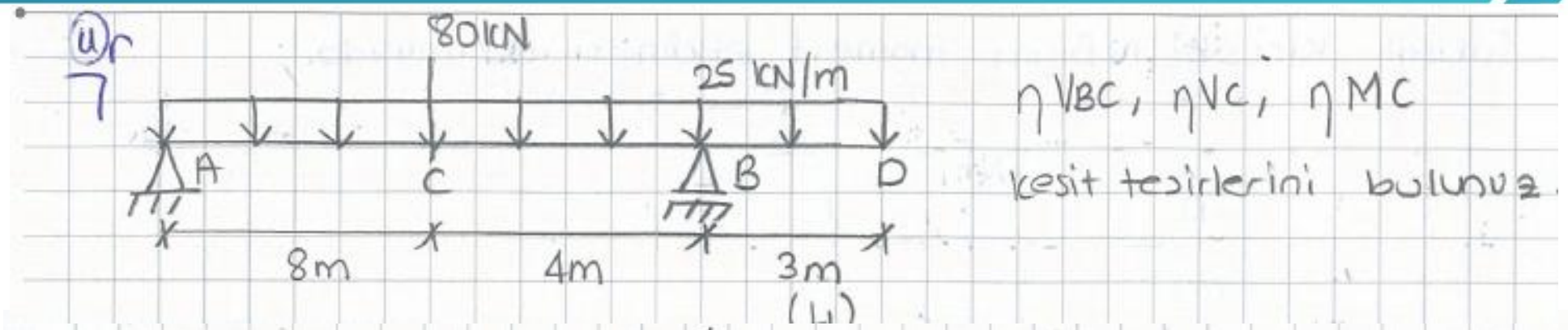
Ör 7

ηV_{BC}, ηV_C, ηM_C kesit tesirlerini bulunuz.

$\frac{a \cdot L}{L} = \frac{8 \cdot 3}{12} = 2$

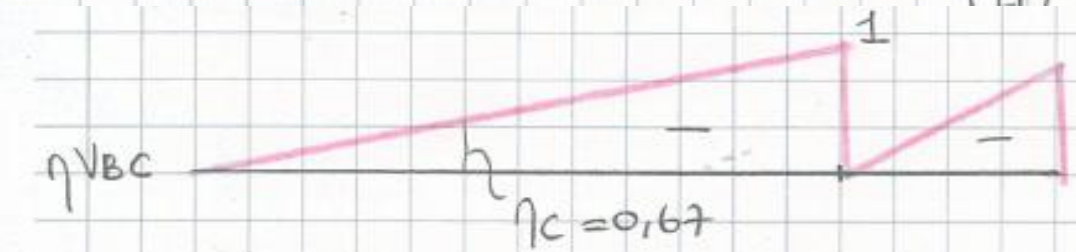
$\frac{a \cdot b}{L} = \frac{8 \cdot 4}{12} = 2{,}67$

$$M_C = 25 \cdot \left(2{,}67 \cdot 12 \cdot \frac{1}{2} - 2 \cdot 3 \cdot \frac{1}{2} \right) + 80 \cdot 2{,}67 \Rightarrow M_C = 539{,}1 \text{ kNm}$$



ⓐ

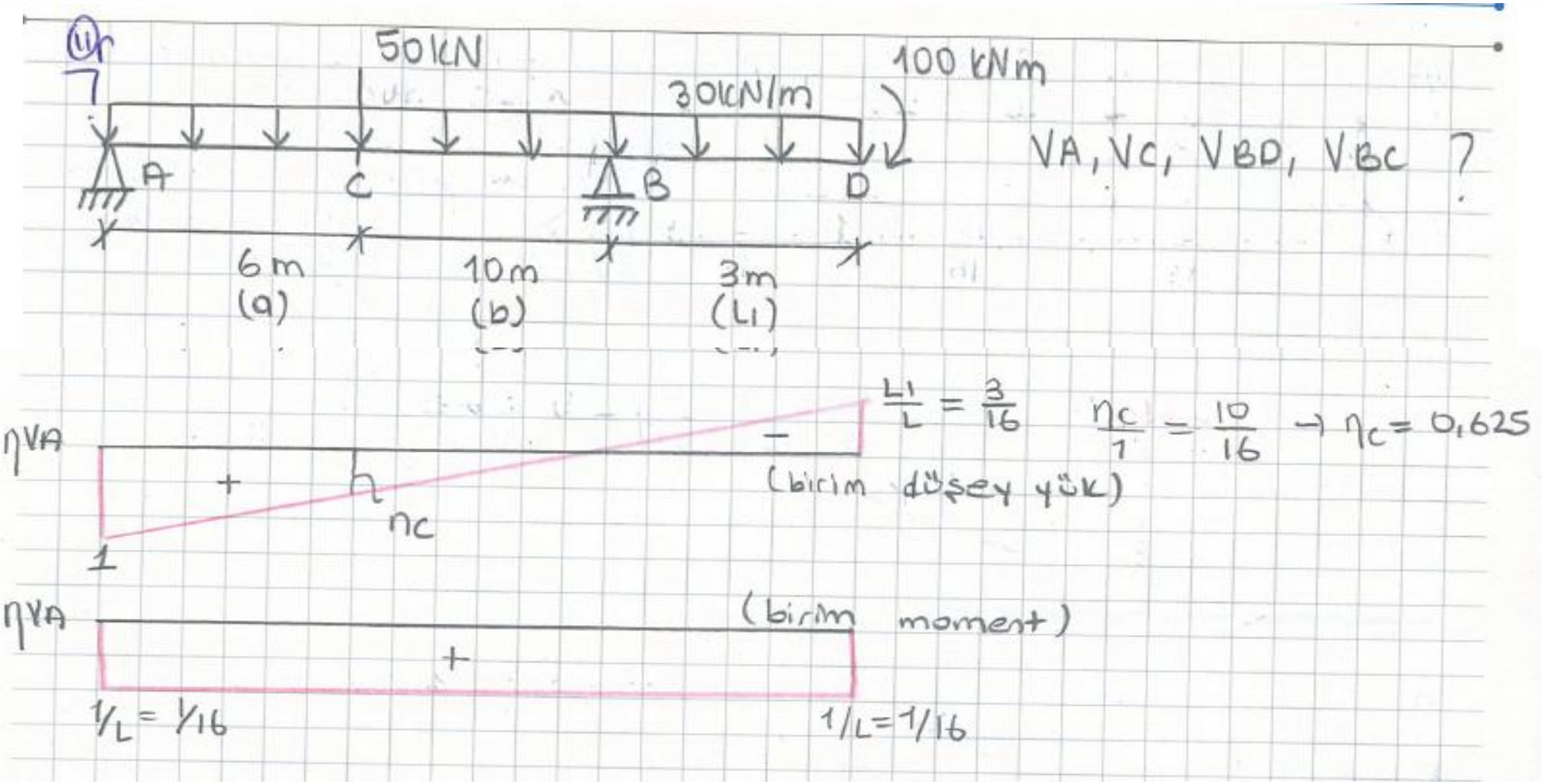
η_{VBC}, η_{VC}, η_{MC} kesit tesirlerini bulunuz.



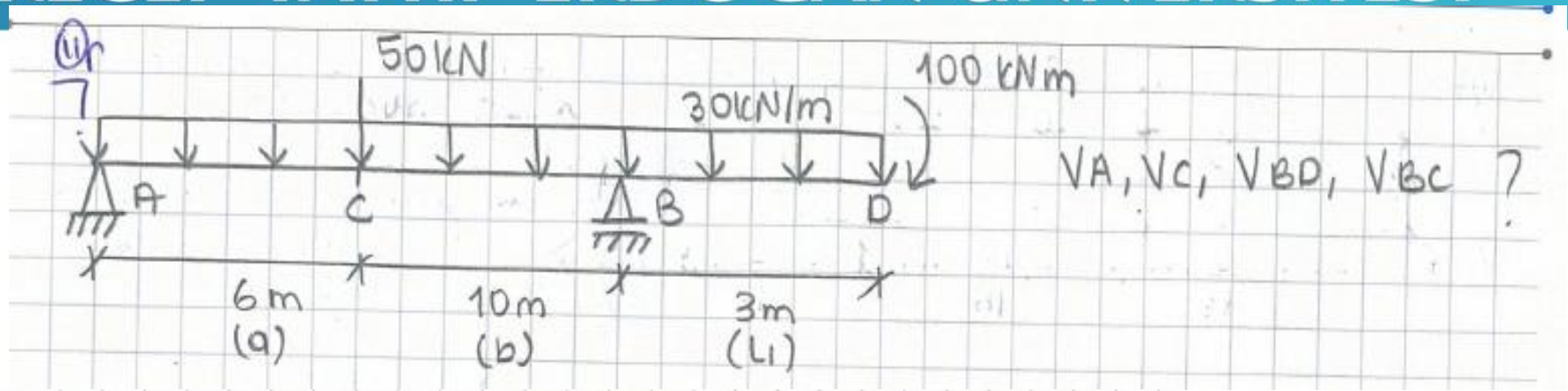
$\frac{L_1}{L} = \frac{3}{12} = 0{,}25$

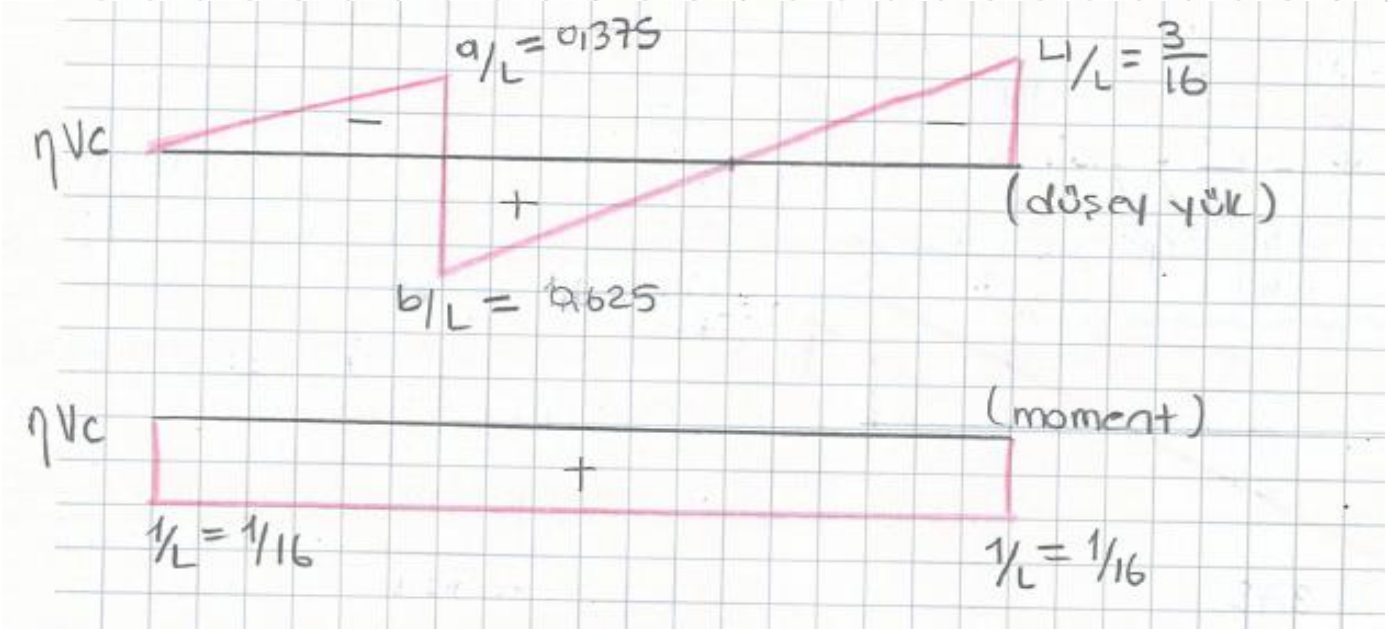
$\frac{\eta_C}{1} = \frac{8}{12} \Rightarrow \eta_C = 0{,}67$

$$V_{BC} = 25 \cdot \left(-1 \cdot 12 \cdot \frac{1}{2} - 0{,}25 \cdot 3 \cdot \frac{1}{2}\right) + 80 \cdot (-0{,}67) \Rightarrow V_{BC} = -212{,}98 \text{ kN}$$



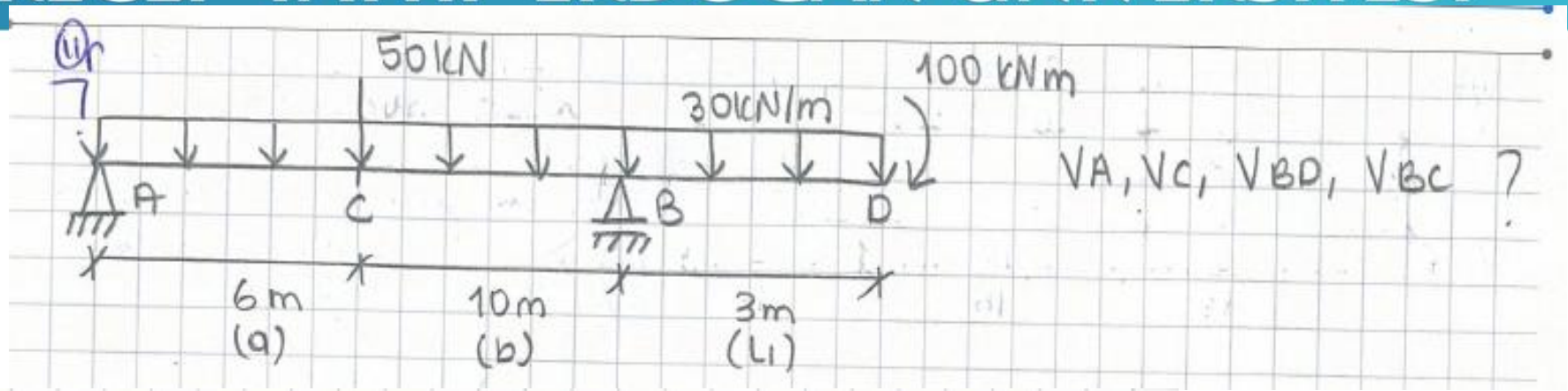
$$VA = 30\left(1 \cdot 16 \cdot \frac{1}{2} - \frac{3}{16} \cdot 3 \cdot \frac{1}{2}\right) + 50 \cdot 0{,}625 - 100 \cdot \frac{1}{16} = 256{,}57\ kN$$

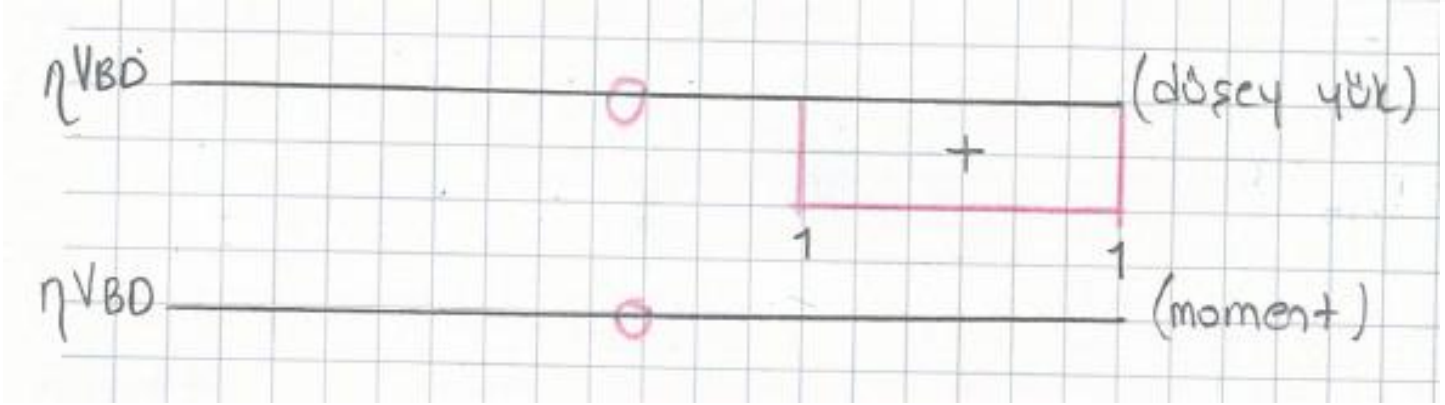




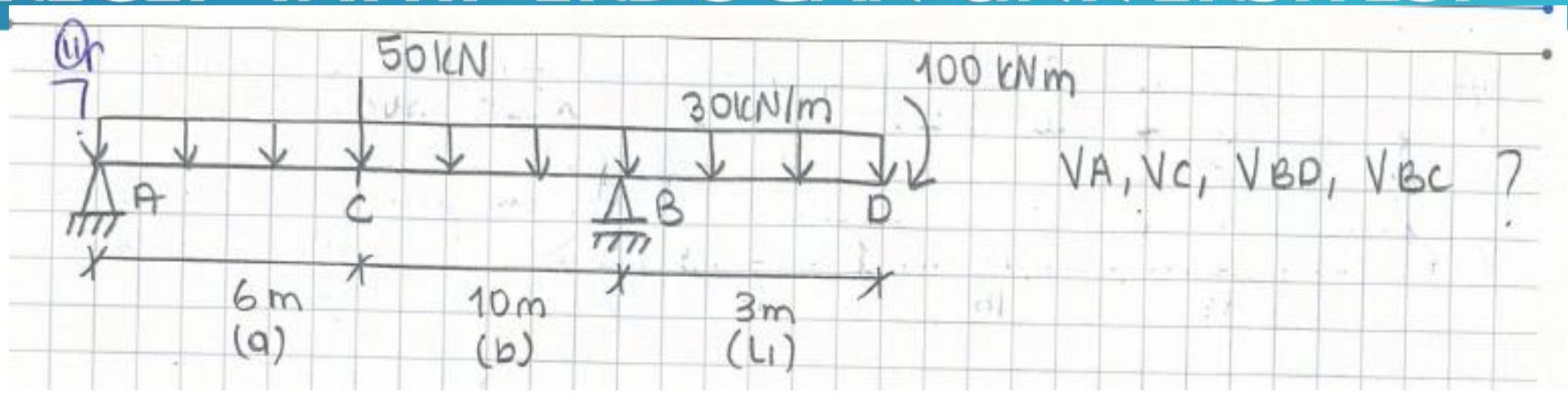
$$V_{CA} = 30\cdot\left(-\frac{6}{16}\cdot 6\cdot\frac{1}{2} + \frac{10}{16}\cdot 10\cdot\frac{1}{2} - \frac{3}{16}\cdot 3\cdot\frac{1}{2}\right) + 50\cdot\frac{10}{16} - 100\cdot\frac{1}{16} = 76{,}57\,kN$$

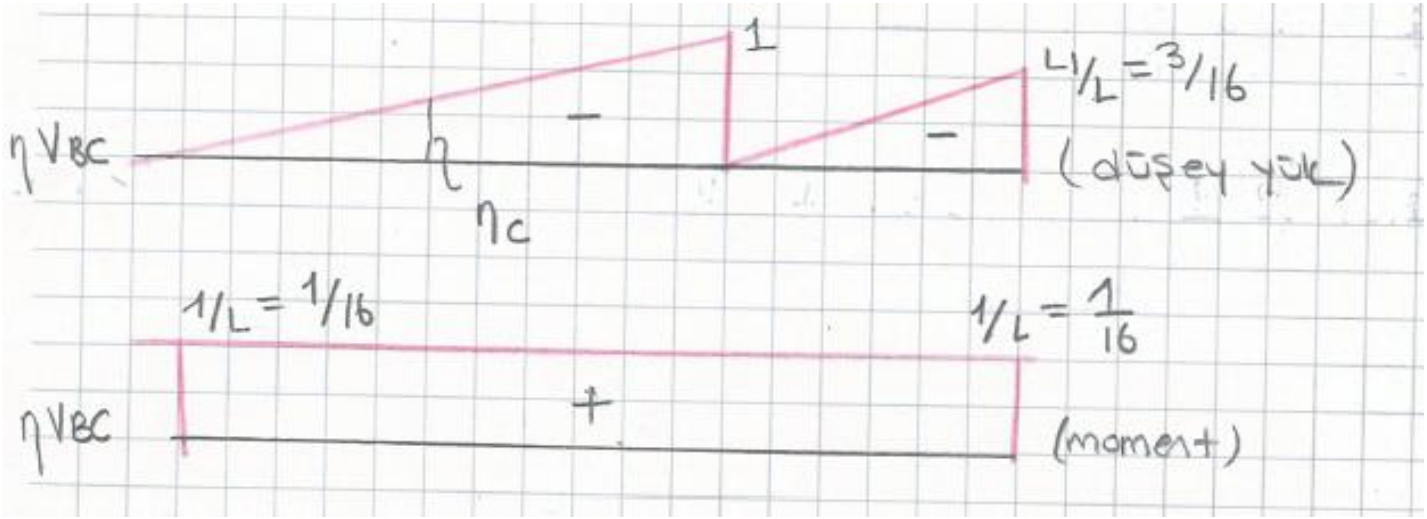
$$V_{CB} = 30\left(-\frac{6}{16}\cdot 6\cdot\frac{1}{2} + \frac{10}{16}\cdot 10\cdot\frac{1}{2} - \frac{3}{16}\cdot 3\cdot\frac{1}{2}\right) + 50\left(-\frac{6}{16}\right) - 100\cdot\frac{1}{16} = 26{,}57\,kN$$



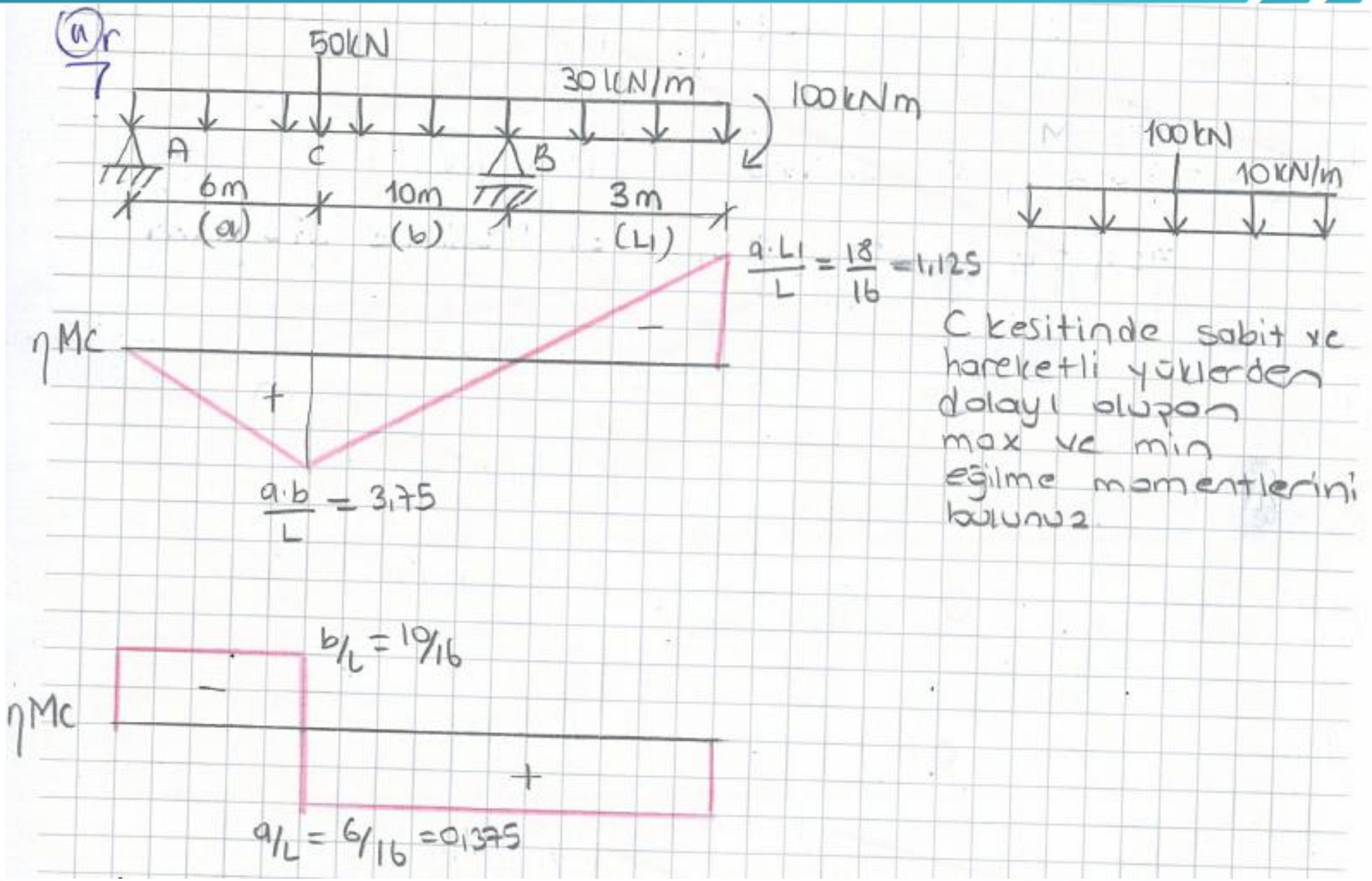


$$V_{BD} = 30 \cdot 1 \cdot 3 = 90 \text{ kN}$$

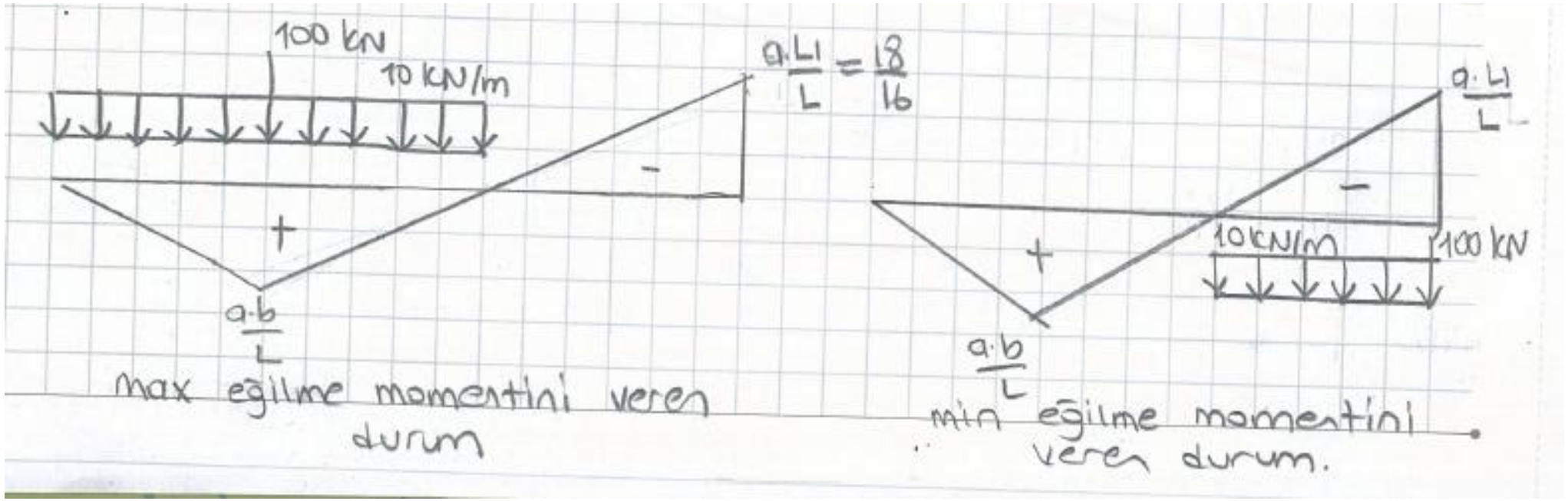




$$V_{BC} = 30\cdot\left(-1\cdot 16\cdot\frac{1}{2} - \frac{3}{16}\,3\cdot\frac{1}{2}\right) + 50\left(-\frac{6}{16}\right) - 100\cdot\left(+\frac{1}{16}\right) = -273{,}43\,kN$$



C kesitinde sabit ve hareketli yüklerden dolayı oluşan max ve min eğilme momentlerini bulunuz


100 kN
10 kN/m
+
-
a·b/L
max eğilme momentini veren durum
a·L1/L = 18/16
a·L1/L
-
+
10kN/m
100 kN
a·b/L
min eğilme momentini veren durum.

Sabit yükten gelen:

$$M_{cg} = 30\left(3{,}75 \cdot 16 \cdot \frac{1}{2} - \frac{18}{16} \cdot 3 \cdot \frac{1}{2}\right) + 50 \cdot (3{,}75) - 100\left(\frac{6}{16}\right) = 999{,}38$$

Hareketli yükten gelen: (max etki)

$$M_{cq} = 10 \cdot \left(3{,}75 \cdot 16 \cdot \frac{1}{2}\right) + 100 \cdot 3{,}75 = 675 \text{ kNm}$$

$$M_{cmax} = M_{cg} + M_{cq}$$

$$M_{cmax} = 999{,}38 + 675 \Rightarrow M_{cmax} = 1674{,}38 \text{ kNm}$$

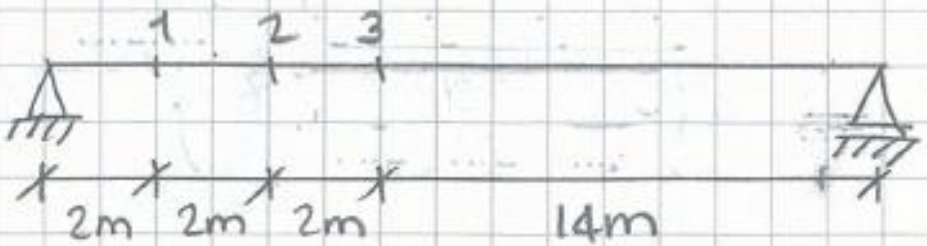
Hareketli yükten gelen: (min etki)

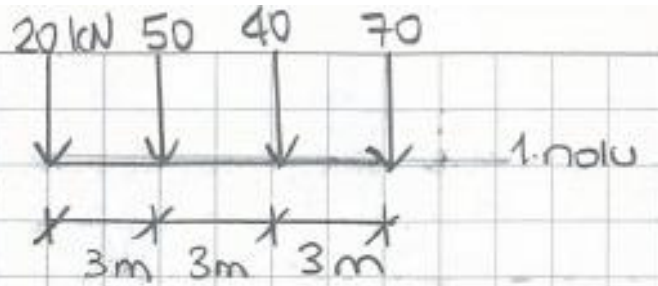
$$M_{cq} = 10 \cdot \left(-\frac{18}{16} \cdot 3 \cdot \frac{1}{2}\right) + 100\left(-\frac{18}{16}\right) = -129{,}38 \text{ kNm}$$

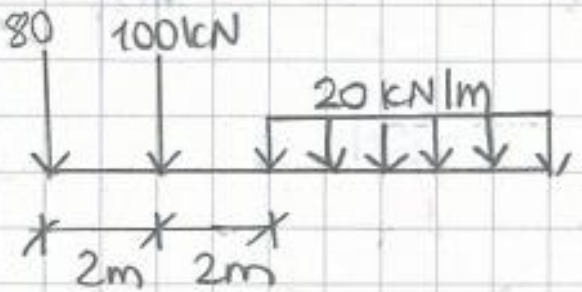
$$M_{cmin} = M_{cg} + M_{cq}$$

$$M_{cmin} = 999{,}38 - 129{,}38 \Rightarrow M_{cmin} = 870 \text{ kNm}$$

Ör 7



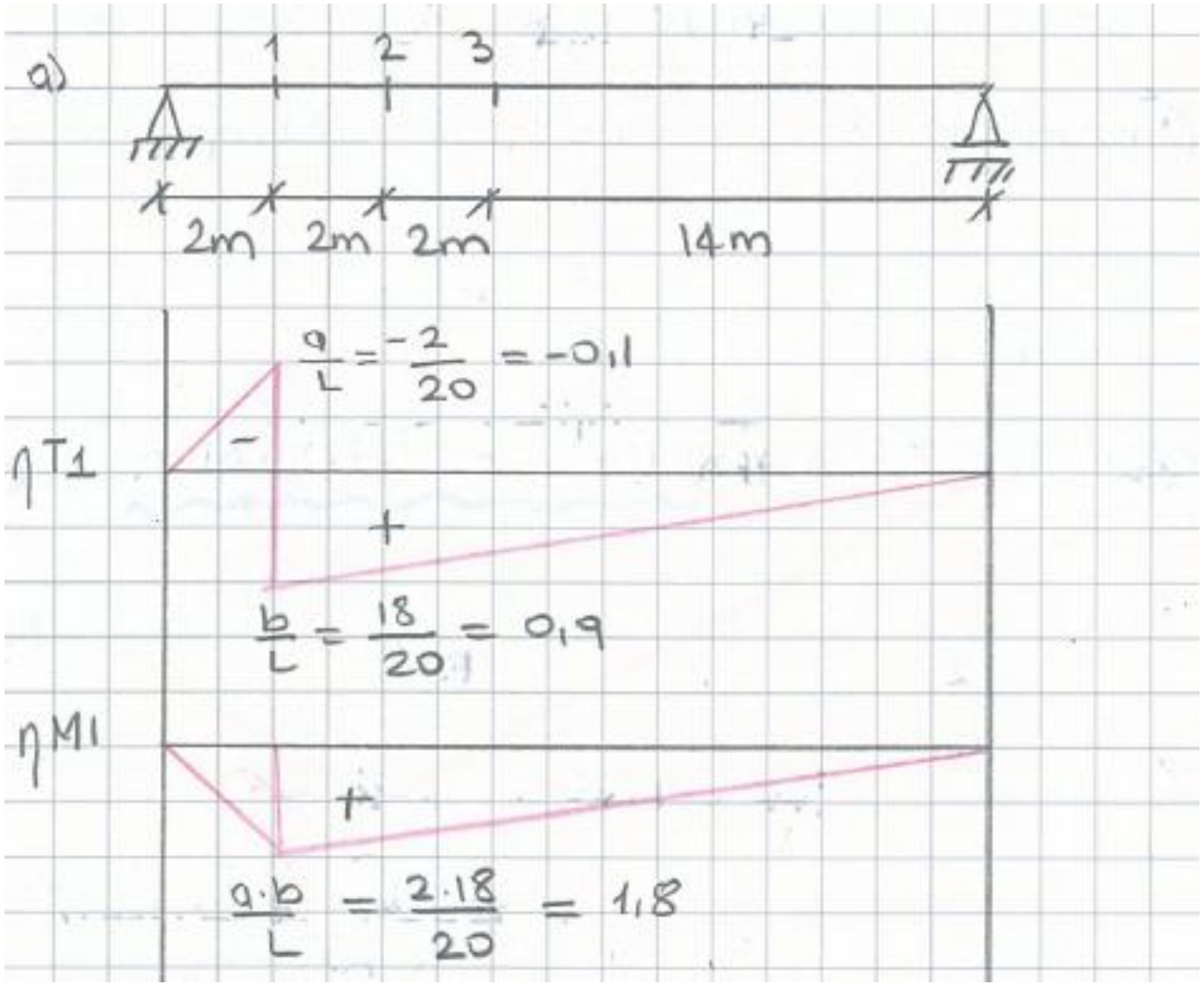


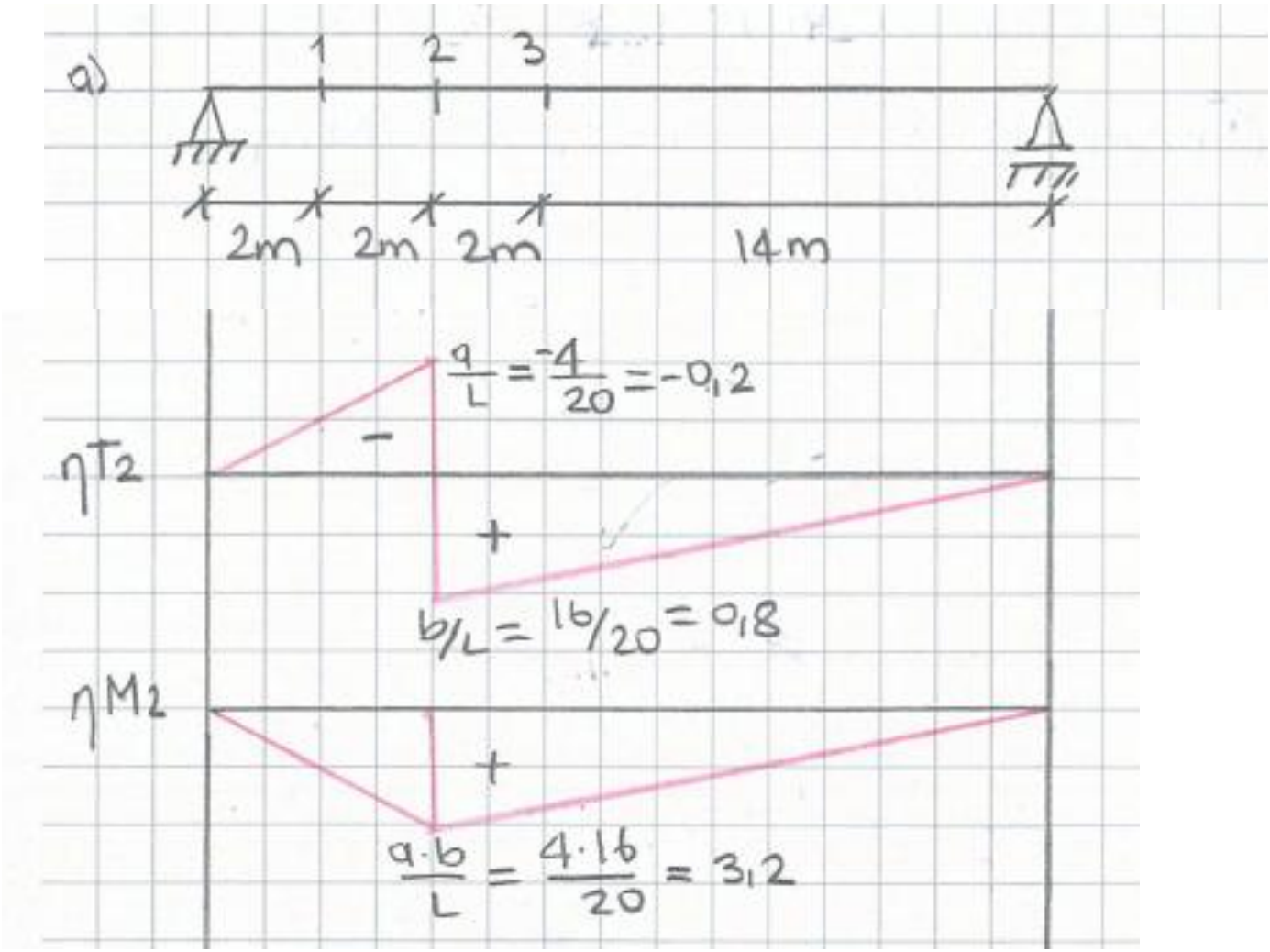


Şekilde verilen basit kiriş üzerine 1 ve 2 nolu yük katarları ayrı ayrı etki etmektedir

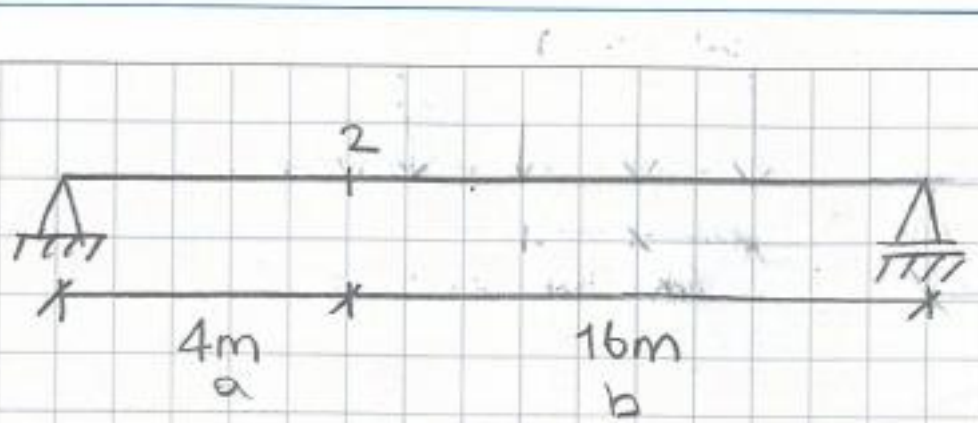
a) 1, 2, 3 kesitlerine ait moment ve kesme kuvveti tesir çizgilerini çiziniz.

b) 1 ve 2 nolu yük katarlarının hangi konumunda 2 kesitinde max moment oluşur ve değeri nedir.


a)
1
2
3
2m
2m
2m
14m
$\frac{a}{L} = \frac{-2}{20} = -0,1$
η_{T1}
−
+
$\frac{b}{L} = \frac{18}{20} = 0,9$
η_{M1}
+
$\frac{a \cdot b}{L} = \frac{2 \cdot 18}{20} = 1,8$

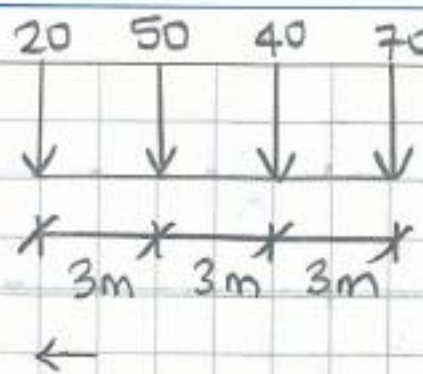

a)
1
2
3
2m
2m
2m
14m
ηT_2
$\frac{a}{L} = \frac{-4}{20} = -0,2$
−
+
$b/L = 16/20 = 0,8$
ηM_2
+
$\frac{a \cdot b}{L} = \frac{4 \cdot 16}{20} = 3,2$

b)





$$\frac{a \cdot b}{L} = \frac{4 \cdot 16}{20} = 3{,}2$$



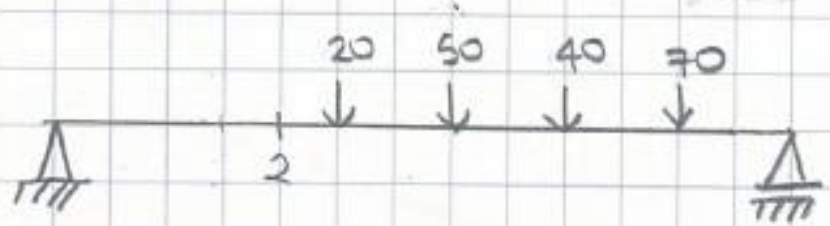
1 nolu:

$\leftarrow$

$\frac{\Sigma Q_a}{\Sigma Q_b} < \frac{q}{b}$ $\leftarrow$ max yapan konumdan biraz önce.

$\frac{\Sigma Q_a}{\Sigma Q_b} > \frac{q}{b}$ $\leftarrow$ max yapan konumdan biraz sonra

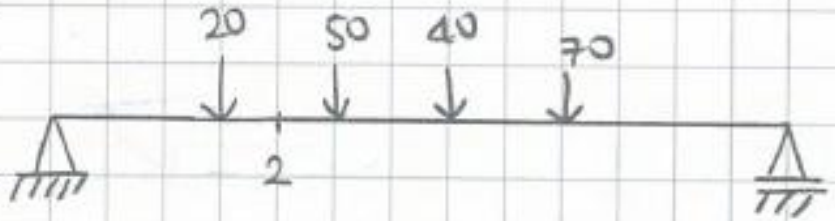
1. konum



$\Sigma Q_a = 0$, $\Sigma Q_b = 180$ kN

$\frac{\Sigma Q_a}{\Sigma Q_b} < \frac{a}{b} \rightarrow \frac{0}{180} < \frac{4}{16} = 0{,}25$ ✗

2. konum



$\Sigma Q_a = 20$, $\Sigma Q_b = 160$ kN

$\frac{\Sigma Q_a}{\Sigma Q_b} = \frac{20}{160} = 0{,}125 < 0{,}25$ ✗

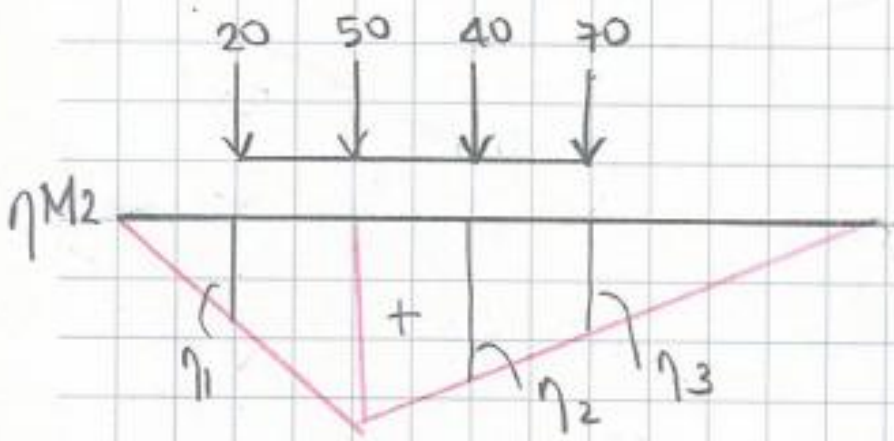
3. konum



$\Sigma Q_a = 70$, $\Sigma Q_b = 110$

$\frac{\Sigma Q_a}{\Sigma Q_b} = \frac{70}{110} = 0{,}63 > 0{,}25$ ✓

max yapan konumdan biraz sonra



$$\frac{a.b}{L} = 3{,}2$$

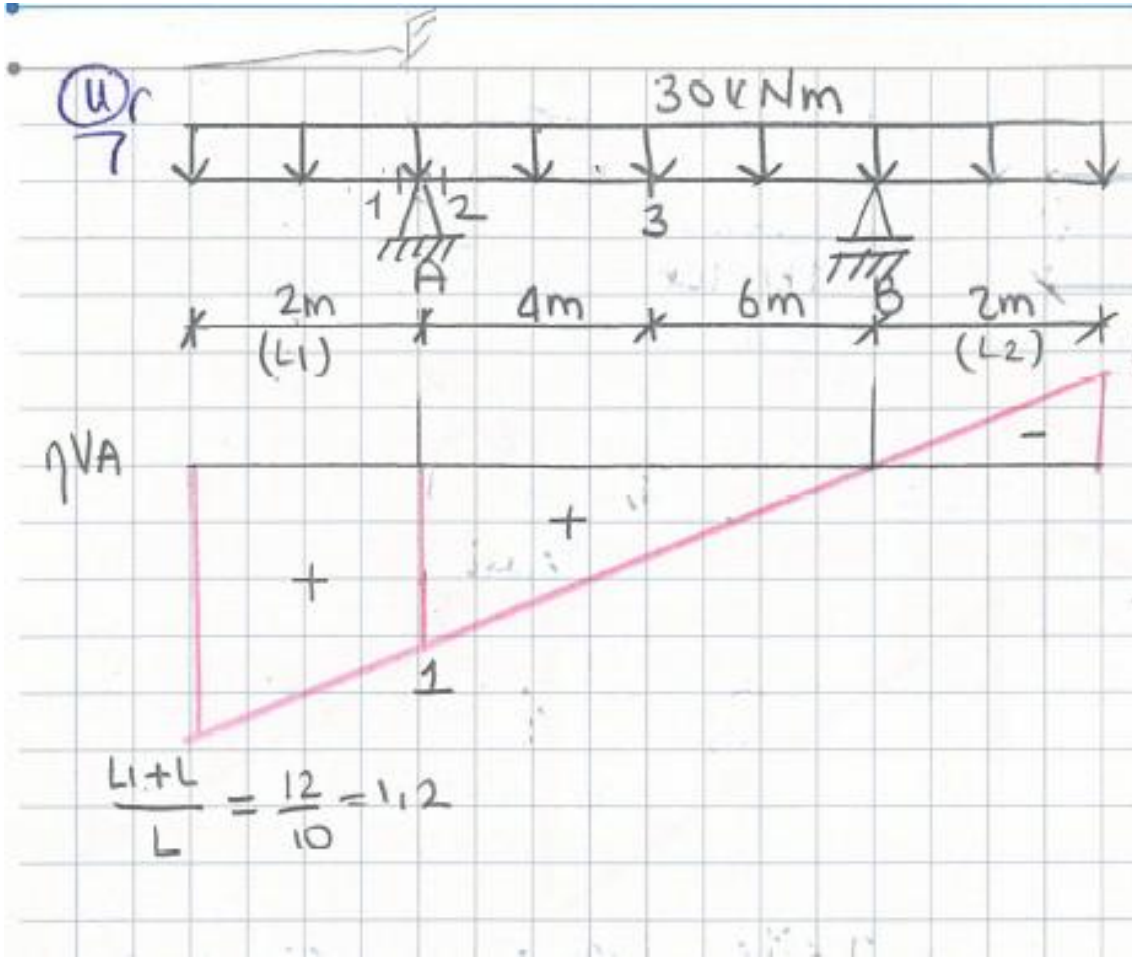
$$\frac{\eta_3}{3{,}2} = \frac{16-6}{16} \Rightarrow \eta_3 = 2$$

$$\frac{\eta_2}{3{,}2} = \frac{16-3}{16} \Rightarrow \eta_2 = 2{,}6$$

$$\frac{\eta_1}{3{,}2} = \frac{4-3}{4} \Rightarrow \eta_1 = 0{,}8$$

$$\max M_2 = 20\eta_1 + 50 \cdot 3{,}2 + 40 \cdot 2{,}6 + 70 \cdot \eta_3 = 420 \text{ kNm.}$$

$$\max M_2 = 80 \cdot 1{,}6 + 100 \cdot 3{,}2 + 20 \cdot 19{,}6 = 840 \text{ kNm}$$

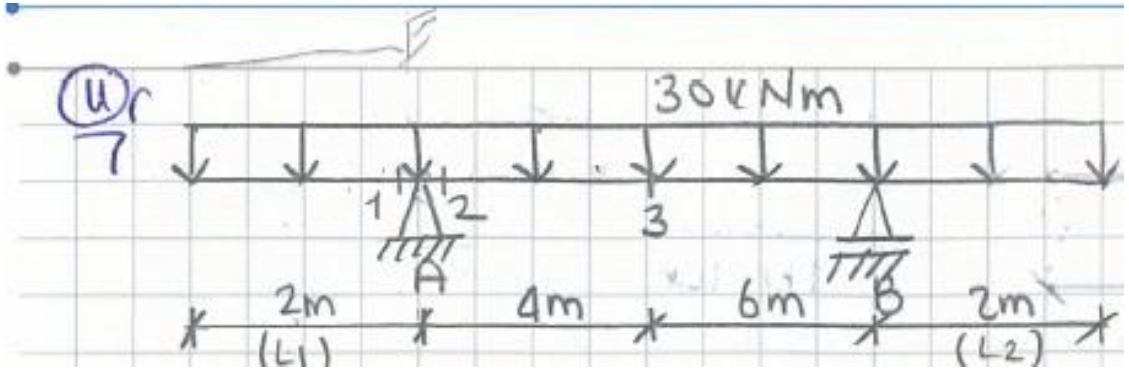


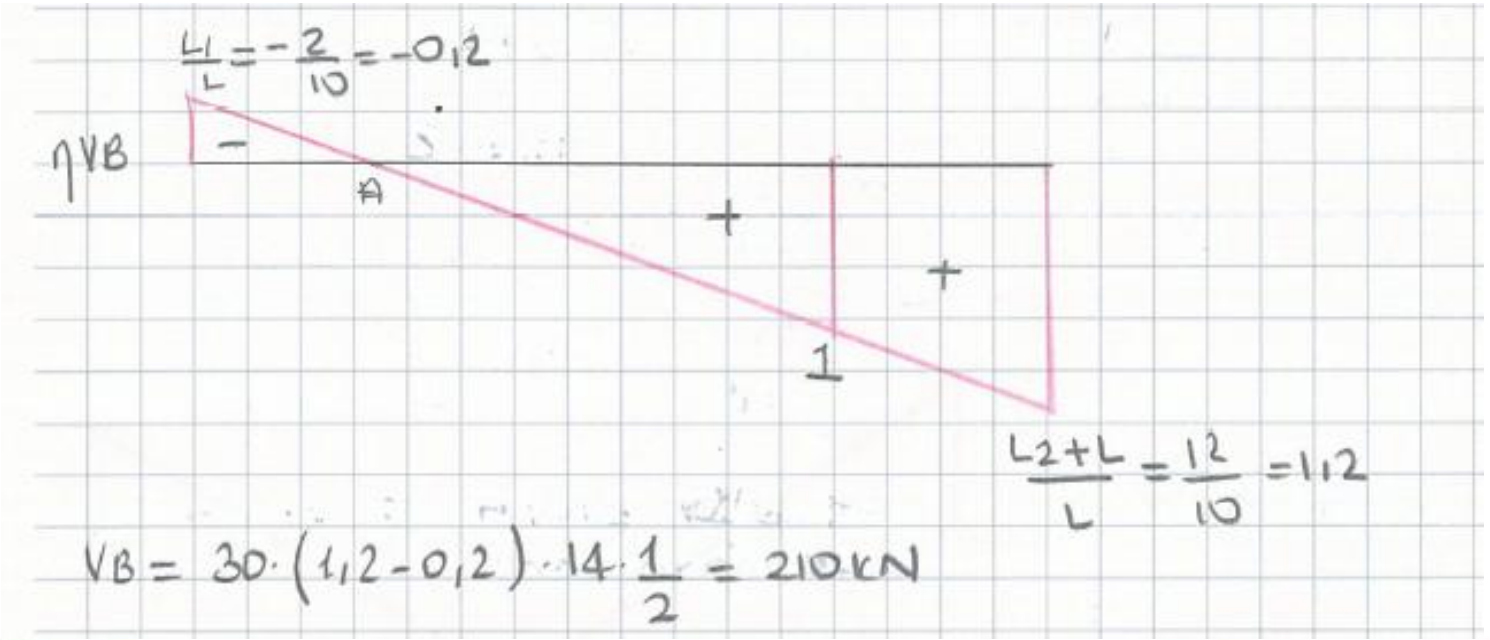
Şekilde verilen sistemin A ve B mesnet tepkilerinin 1, 2, 3 kesitlerindeki moment ve kesme kuvveti değerlerini tesir çizgilerinden yararlanarak hesaplayınız.

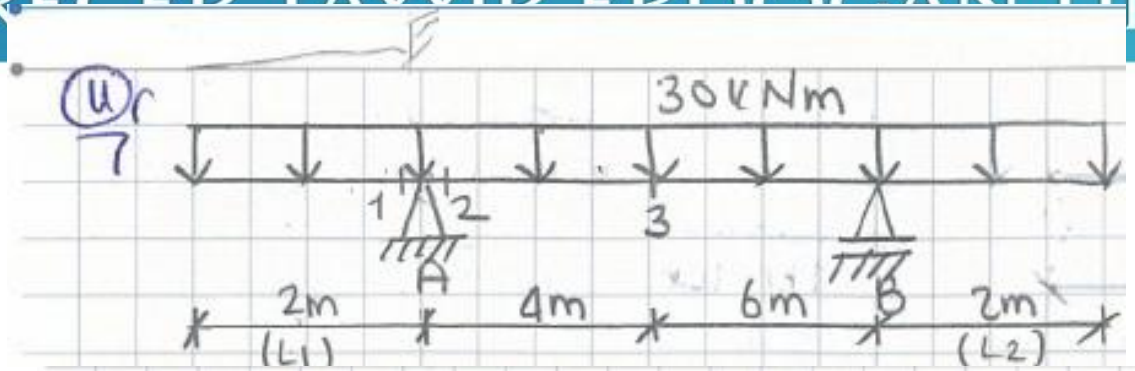
$$V_A = 30 \cdot \left(1{,}2 - 0{,}12\right) \cdot 14 \cdot \frac{1}{2} = 210\,kN$$

Kontrol $\curvearrowright \Sigma M_B = 0$

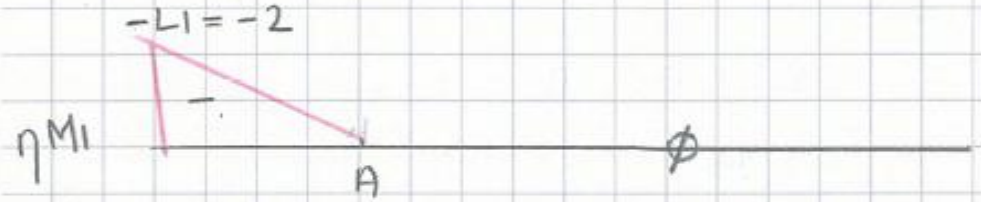
$V_A \cdot 10 - 30 \cdot 14 \cdot 5 = 0 \rightarrow V_A = 210$


Ör
7
30 kNm
1
2
3
A
B
2m
(L1)
4m
6m
2m
(L2)


L1/L = -2/10 = -0,2
ηVB
−
A
+
+
1
L2+L/L = 12/10 = 1,2
VB = 30·(1,2-0,2)·14·1/2 = 210 kN



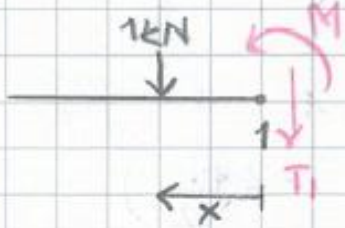




$$T_1 = 30 \cdot (-1 \cdot 2) = -60\ kN$$

$$M_1 = 30 \cdot \left(-2 \cdot 2 \cdot \frac{1}{2}\right) = -60\ kN$$

1kN'luk kuvvet 1 kesitinin sandayken



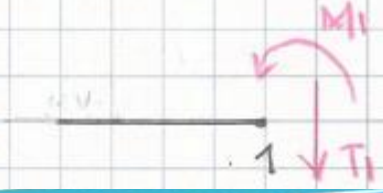
$\Sigma Y = 0 \quad T_1 = -1\ kN$

$\Sigma M_1 = 0$

$M_1 + 1 \cdot x = 0$

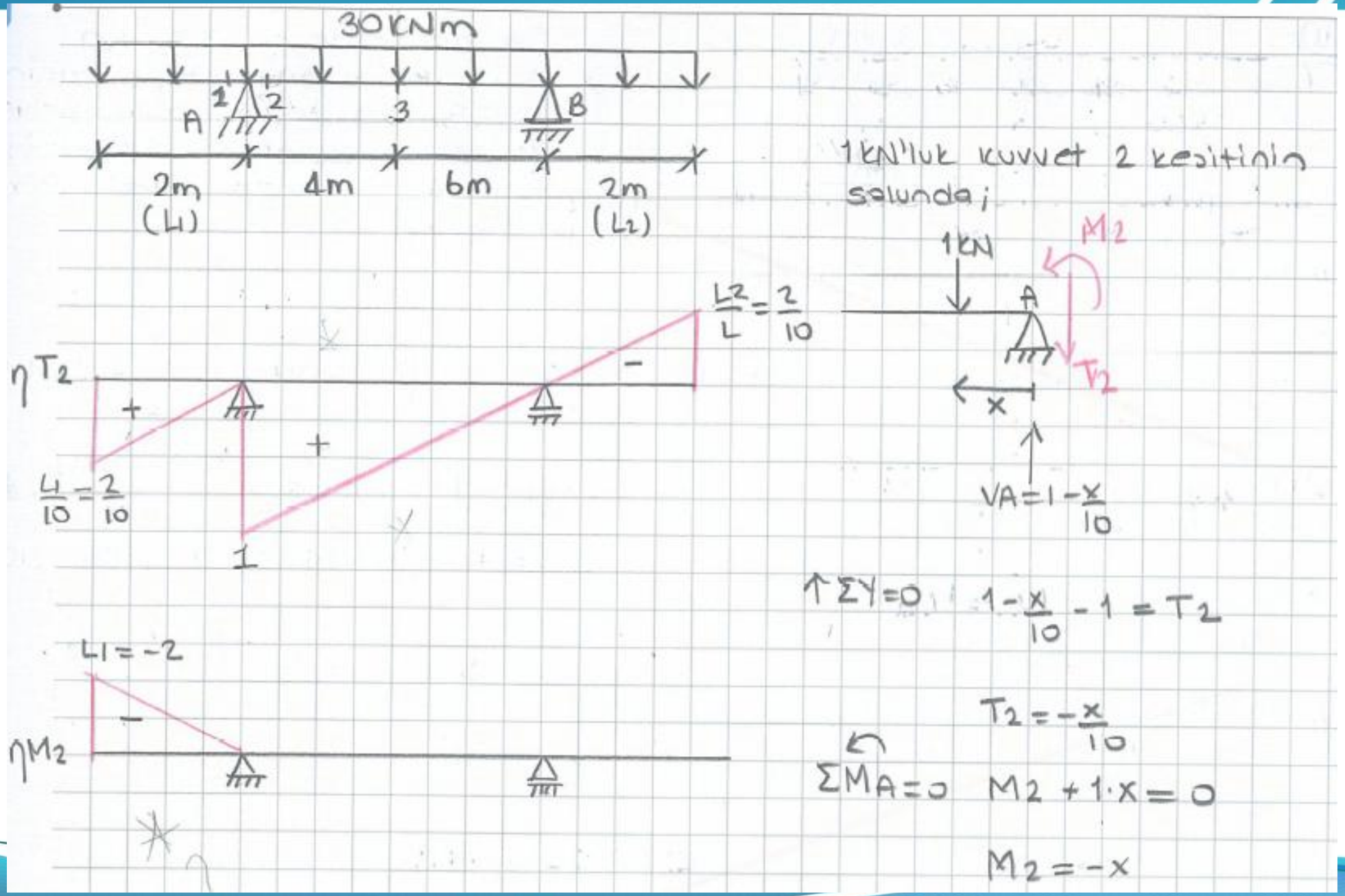
$M_1 = -x$

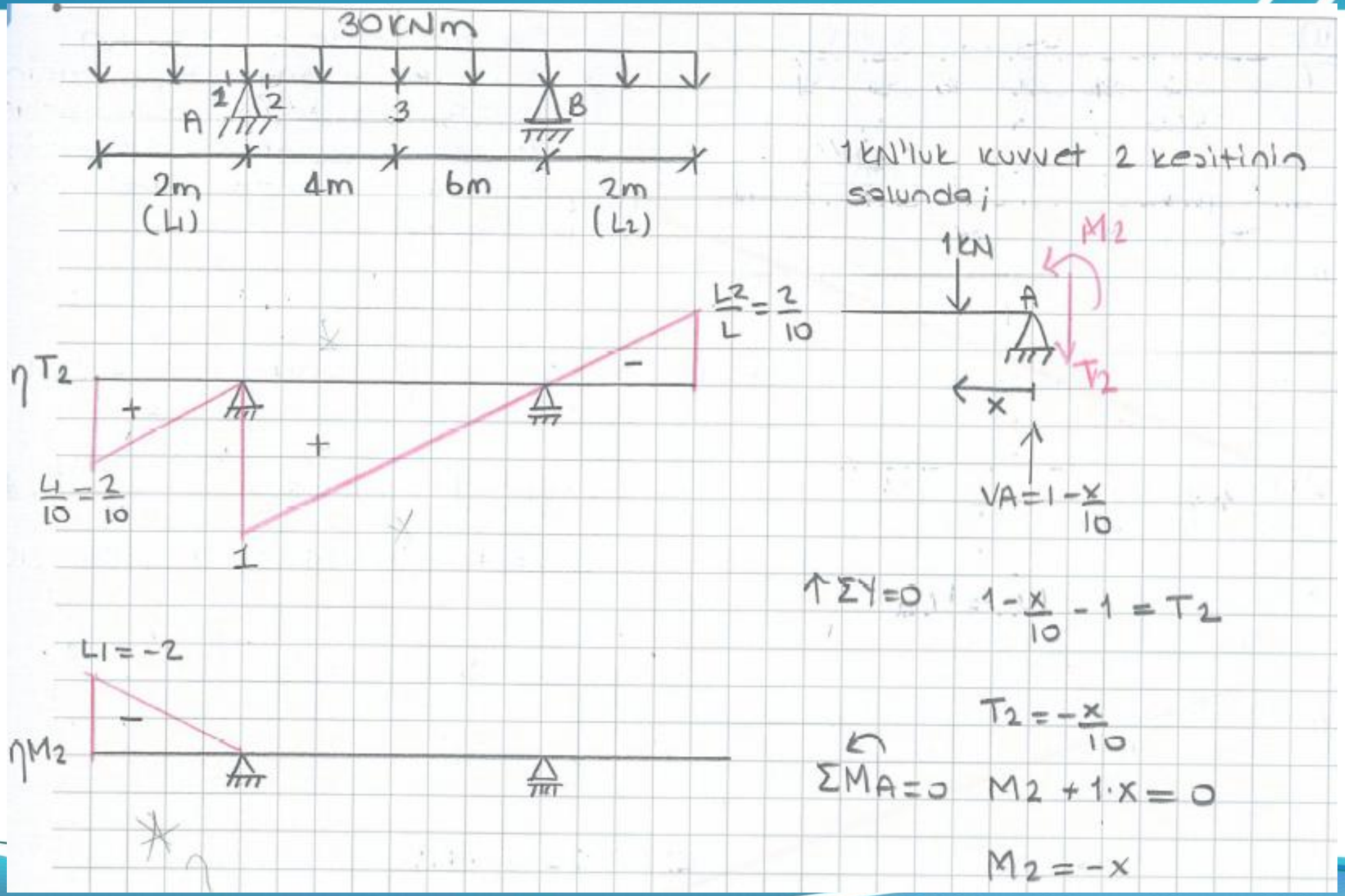
1kN'luk kuvvet 1 kesitinin sağındayken



$\Sigma Y = 0 \quad T_1 = 0$

$\Sigma M_1 = 0 \quad M_1 = 0$


30 kNm
A
1'
1
2
3
B
2m
(L1)
4m
6m
2m
(L2)
η T2
+
+
−
L1/10 = 2/10
1
L2/L = 2/10
η M2
L1 = −2
−
1 kN'luk kuvvet 2 kesitinin solunda;
1kN
A
M2
T2
x
VA = 1 − x/10
↑ΣY=0 1 − x/10 − 1 = T2
T2 = −x/10
↶ΣMA=0 M2 + 1·x = 0
M2 = −x


30 kNm
A
1'
1
2
3
B
2m
(L1)
4m
6m
2m
(L2)
η T2
+
+
−
L1/10 = 2/10
1
L2/L = 2/10
η M2
L1 = −2
−
1 kN'luk kuvvet 2 kesitinin solunda;
1kN
A
M2
T2
x
VA = 1 − x/10
↑ΣY=0 1 − x/10 − 1 = T2
T2 = −x/10
↶ΣMA=0 M2 + 1·x = 0
M2 = −x

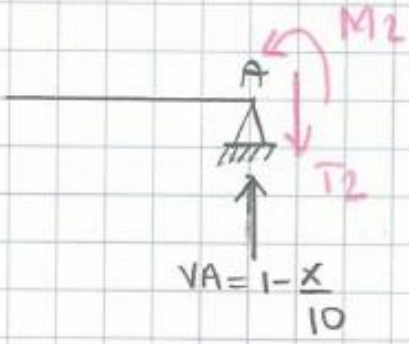
$$T_2 = 30\cdot\left(0{,}2\cdot 2\cdot\frac{1}{2} + 1\cdot 10\cdot\frac{1}{2} - 0{,}2\cdot 2\cdot\frac{1}{2}\right)$$

$$T_2 = 150 \text{ kN}$$

$$M_2 = 30\cdot\left(-2\cdot 2\cdot\frac{1}{2}\right)$$

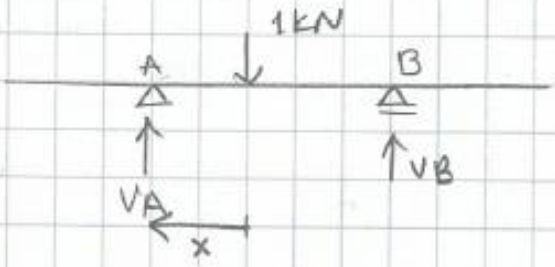
$$M_2 = -60 \text{ kNm}$$

1 kN'luk kuvvet 2 kesitinin sağındayken;



$\uparrow \Sigma Y = 0 \quad T_2 = 1 - \frac{x}{10}$

$\circlearrowleft \Sigma M_A = 0 \quad M_2 = 0$

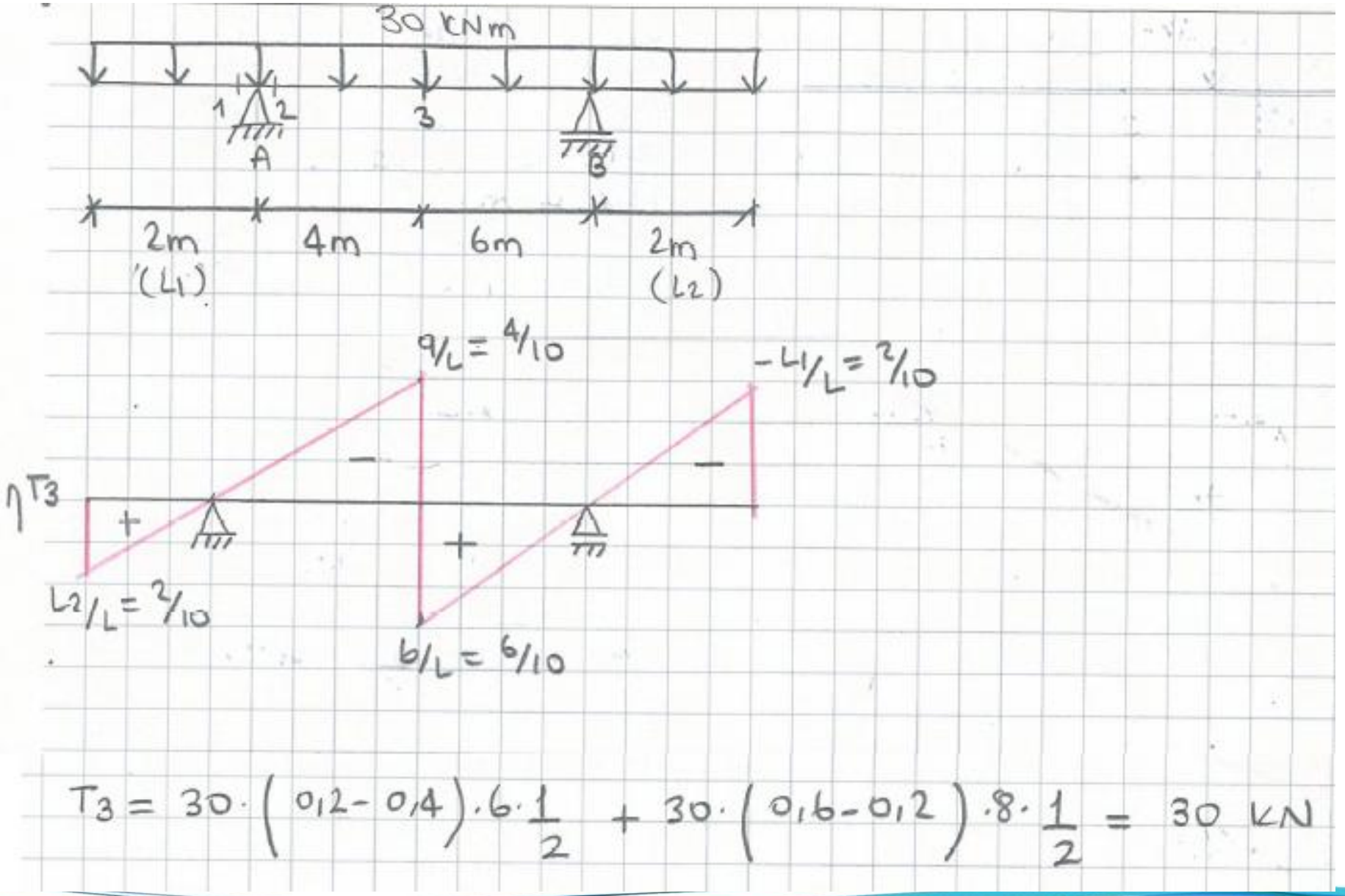


$\circlearrowleft \Sigma M_A = 0 \quad V_B \cdot 10 = 1 \cdot x$

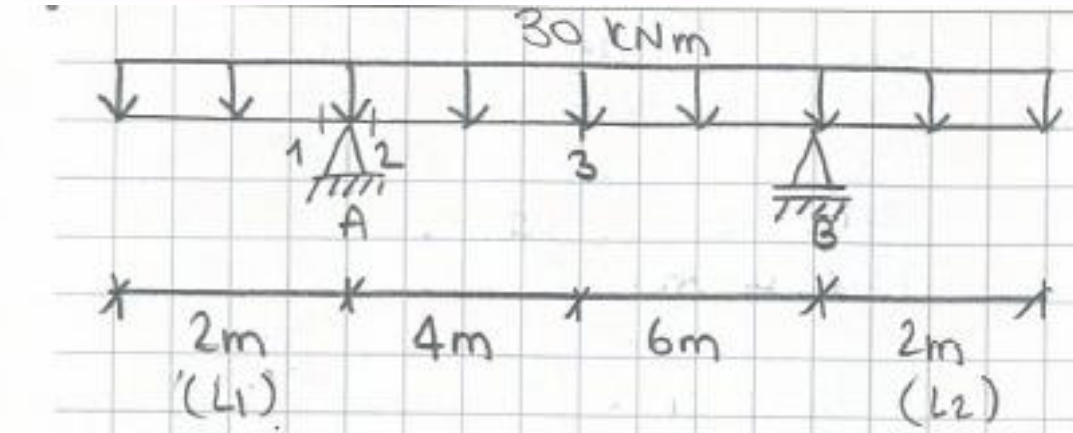
$$V_B = \frac{x}{10}$$

$\circlearrowright \Sigma M_B = 0 \quad V_A \cdot 10 = 1\cdot(L-x)$

$$V_A = \frac{L-x}{10} \quad (L=10)$$

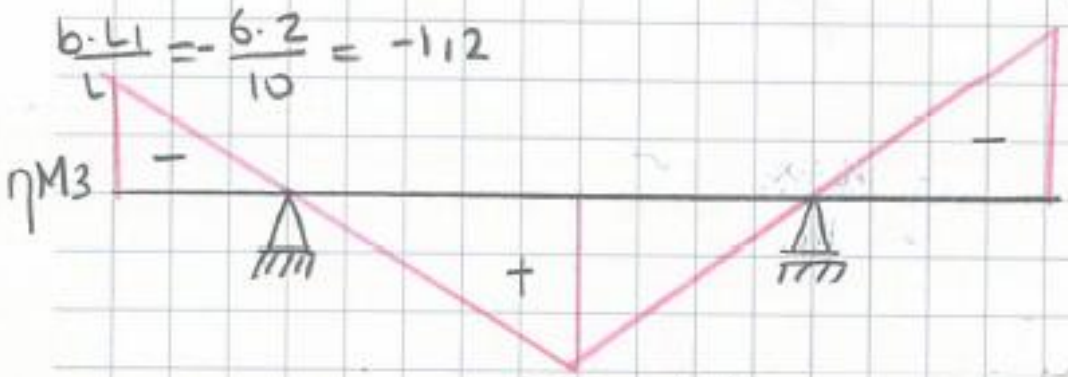


$$T_3 = 30 \cdot (0{,}2 - 0{,}4) \cdot 6 \cdot \frac{1}{2} + 30 \cdot (0{,}6 - 0{,}2) \cdot 8 \cdot \frac{1}{2} = 30 \text{ kN}$$



$\frac{b \cdot L_1}{L} = -\frac{6 \cdot 2}{10} = -1{,}2$

$\frac{a \cdot L_2}{L} = \frac{4 \cdot 2}{10} = -0{,}8$



$\frac{a \cdot b}{L} = \frac{6 \cdot 4}{10} = 2{,}4$

$$M3 = 30 \cdot \left(-1{,}2 \cdot 2 \cdot \frac{1}{2} + 2{,}4 \cdot 10 \cdot \frac{1}{2} - 0{,}8 \cdot 2 \cdot \frac{1}{2} \right) = 300 \text{ kNm}$$

KAYNAKLAR

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