RECEP TAYYİP ERDOĞAN ÜNİVERSİTESİ
Mühendislik ve Mimarlık Fakültesi
İnşaat Mühendisliği Bölümü

INS313 YAPI STATİĞİ-I

Dr. Öğr. Üyesi Zeliha TONYALI

"11H. BASİT KİRİŞLERDE TESİR ÇİZGİLERİ"

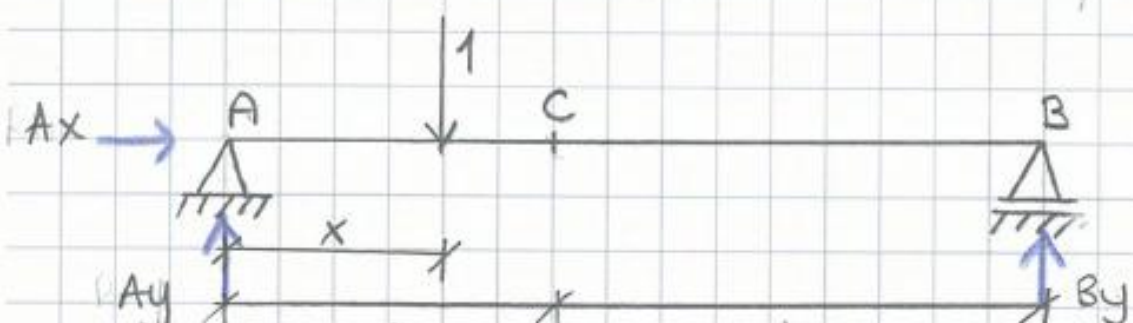
TESİR ÇİZGİSİ

Tesir fonksiyonu sistem üzerinde belirli bir doğrultuda hareket eden 1 br'lik bir kuvvetten dolayı belirli bir kesitteki statik büyüklüğün (mesnet tepkisi, kesme kuvveti, eğilme momenti, normal kuvvet) değişimini gösterir.

Tesir çizgisi sistem üzerinde hareket eden 1 birimlik kuvvetten, dolayı kesitte meydana gelen büyüklük, 1 brmlik yük hizasında ordinat olarak alınarak hesaplanır.

İzostatik sistemlerde kesit tesirleri ve mesnet tepkilerinin tesir çizgileri doğrusaldır.

1 Birimlik düşey kuvvet etkimesi durumunda;



$\Sigma X = 0 \rightarrow$ $AX = 0$

$\curvearrowright \Sigma M_B = 0$ $\quad Ay \cdot L = 1(L-x)$

$$Ay = \frac{(L-x)}{L}$$

$x=0$ için $Ay = 1$

$x=L$ " $Ay = 0$



$\curvearrowleft \Sigma M_A = 0$ $\quad By \cdot L = 1 \cdot x$

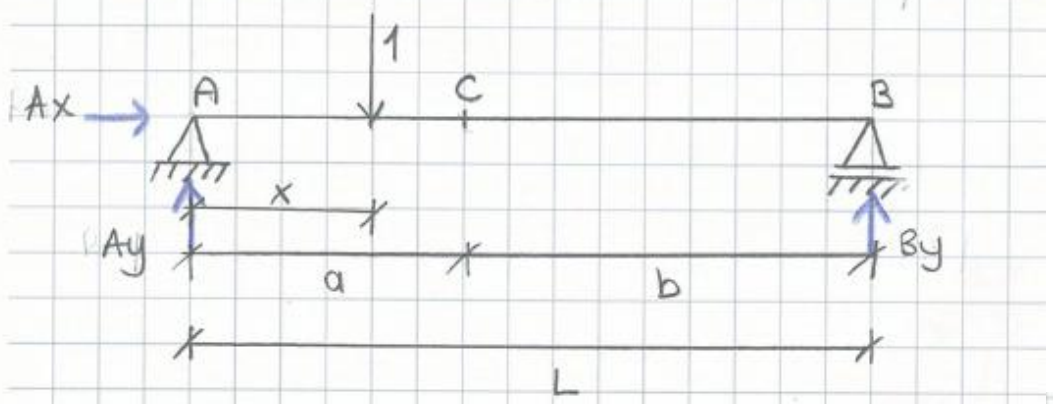
$$By = \frac{x}{L}$$

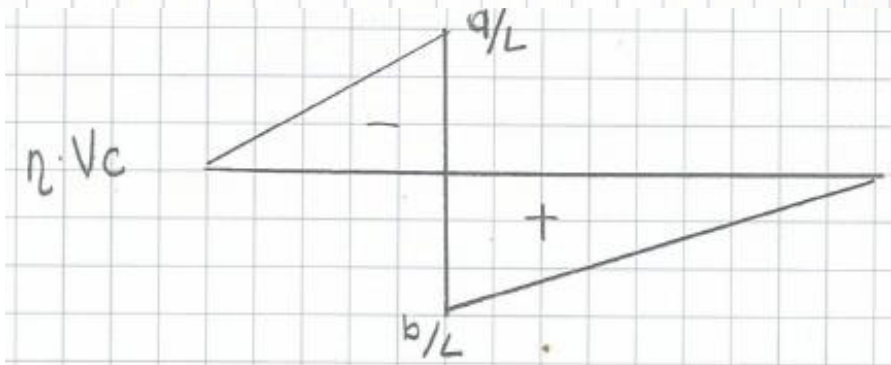
$x=0$ için $By = 0$

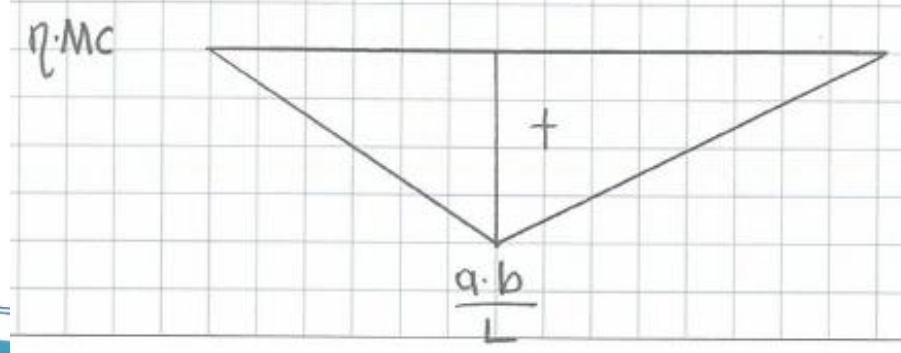
$x=L$ için $By = 1$



1 Birimlik düşey kuvvet etkimesi durumunda





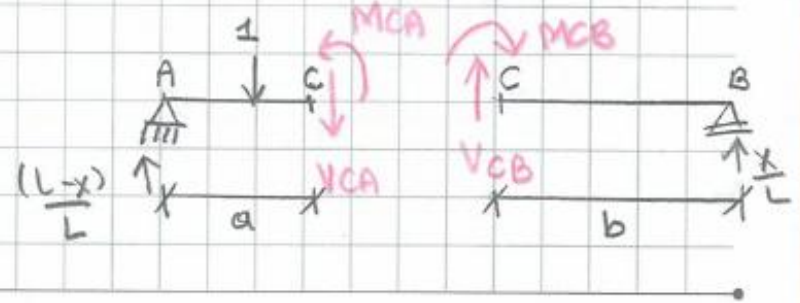


* C kesitindeki kesme kuvveti ve eğilme momenti tesir çizgisi

A-C arası → $0 < x < a$

C-B arası → $a < x < L$

Yük A-C arasında ise;



Yük A-C arasında ise C kesitinden kesim yapılır ve daha az kuvvet bulunduğu için sağdakalan C-B sistemi üzerinden denge denklemleri yazılır.

$\Sigma Y = 0 \qquad V_{CB} + B_y = 0 \rightarrow V_{CB} = -B_y = \frac{-x}{L}$

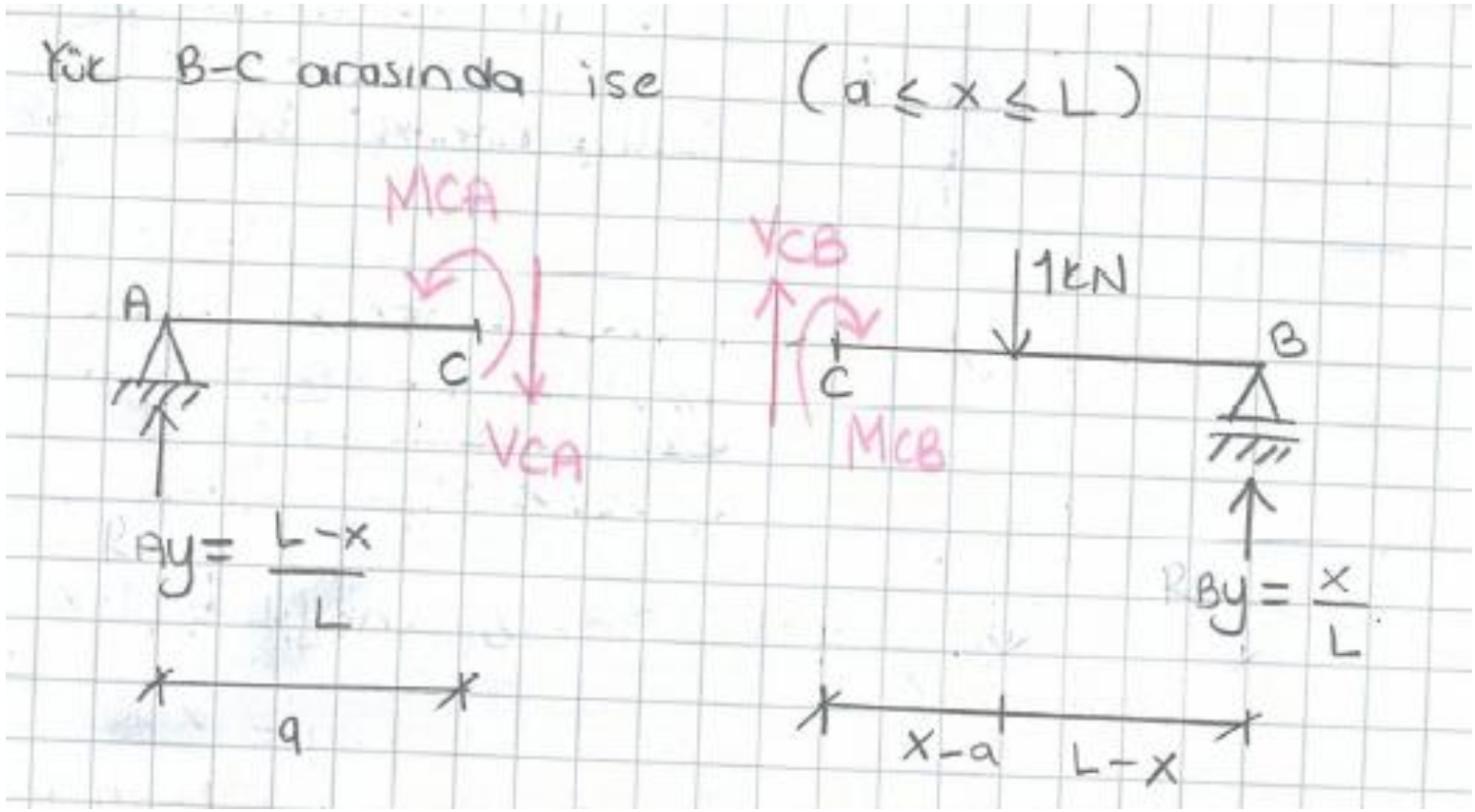
$x = 0$ için $V_{CB} = 0$

$x = a$ " $V_{CB} = -\frac{a}{L}$

$\Sigma M_C = 0 \qquad B_y \cdot b = M_{CB} \rightarrow M_{CB} = b \cdot \frac{x}{L}$

$x = 0$ için $M_{CB} = 0$

$x = a$ " $M_{CB} = \frac{a \cdot b}{L}$


Yük B-C arasında ise (a ≤ x ≤ L)
MCA
VCA
A
C
Ay = L-x / L
q
VCB
MCB
1kN
C
B
By = x / L
x-a
L-x

Solda kalan A-C sistemi üzerinden denge denklemleri yazılırsa;

$\Sigma Y = 0$ $\quad A_y = V_{CA} \rightarrow V_{CA} = \frac{L-x}{L}$

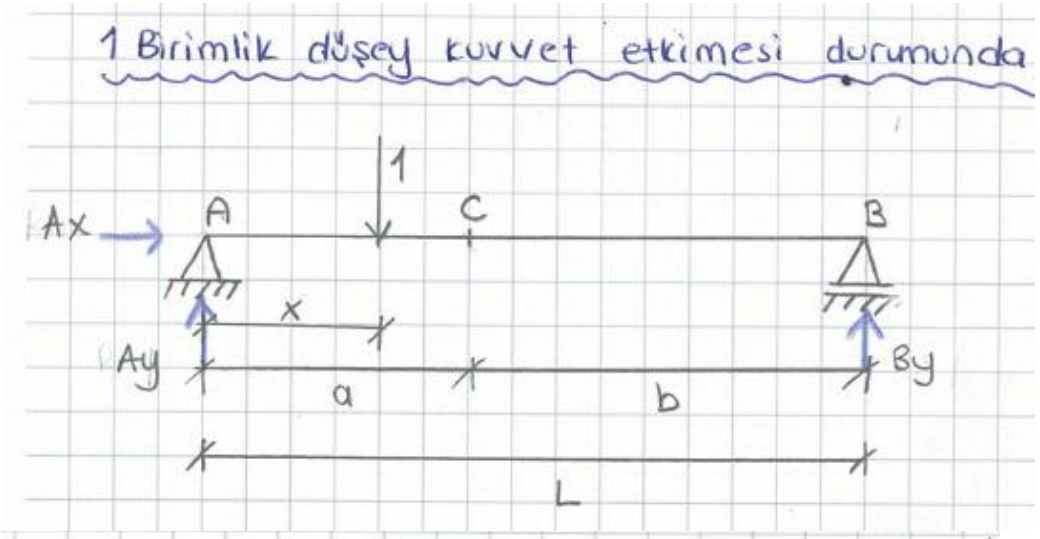
$x = a$ için $\quad V_{CA} = \frac{L-a}{L} = \frac{b}{L}$

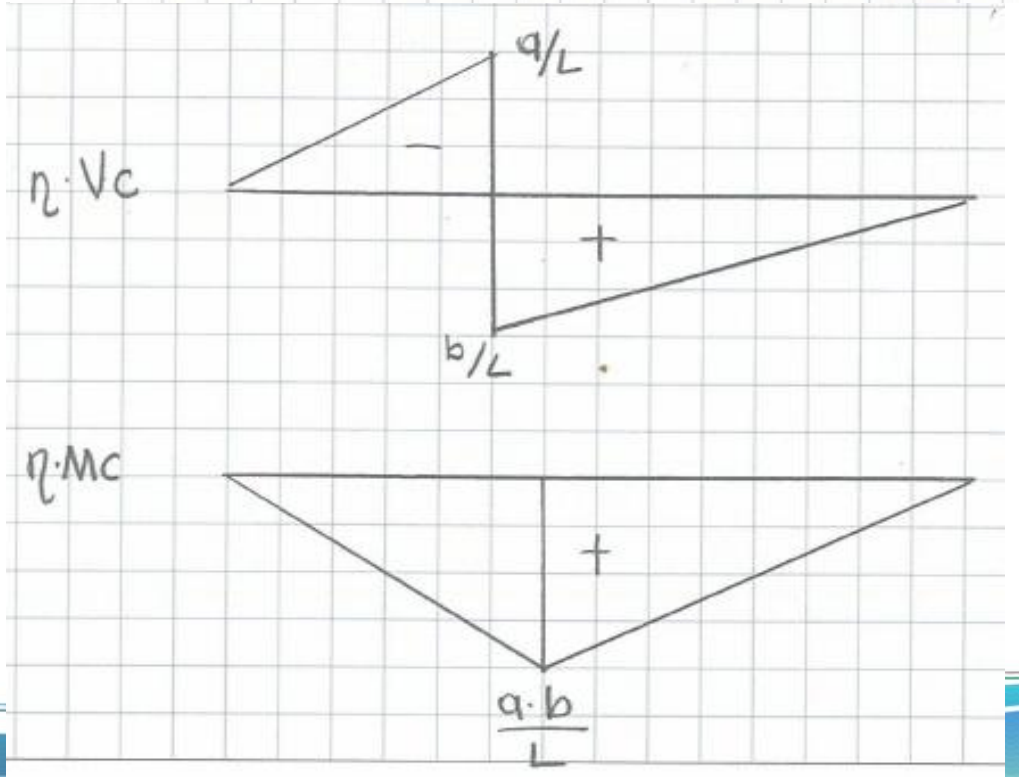
$x = L$ için $\quad V_{CA} = 0$

$\overset{\curvearrowright}{\Sigma M_C = 0}$ $\quad A_y \cdot a = M_{CA} \rightarrow M_{CA} = a \cdot \frac{(L-x)}{L}$

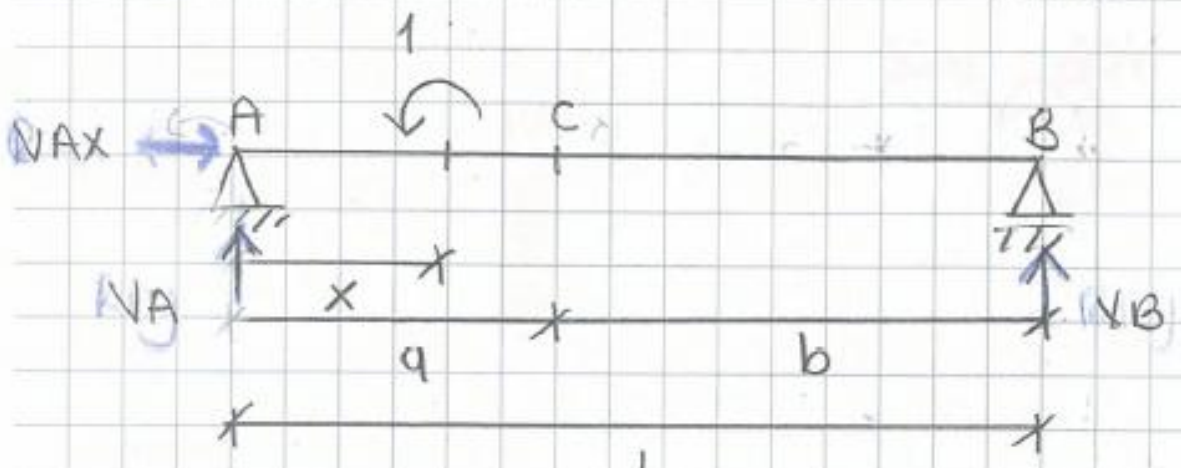
$x = a$ için $\quad M_{CA} = \frac{a \cdot b}{L}$

$x = L$ için $\quad M_{CA} = 0$


1 Birimlik düşey kuvvet etkimesi durumunda
1
Ax
A
C
B
x
Ay
By
a
b
L


q/L
η·Vc
−
+
b/L
η·Mc
+
a·b
L

1 Birimlik moment etkimesi durumunda;



Tüm sistemden $\Sigma X = 0$

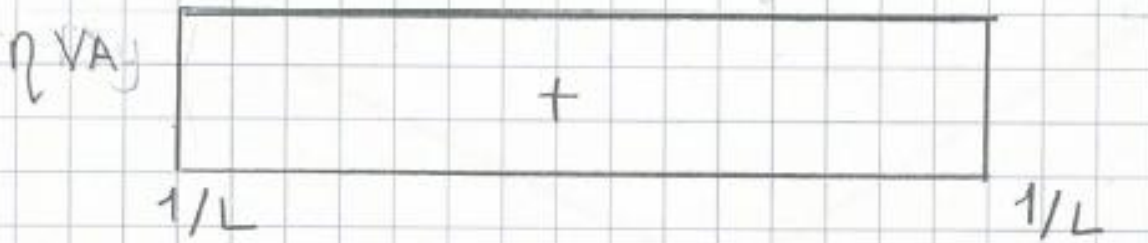
$NAX = 0$

$\Sigma M_B = 0 \quad 1 = V_A \cdot L$

$V_A = \frac{1}{L}$

$\Sigma M_A = 0 \quad 1 + V_B L = 0$

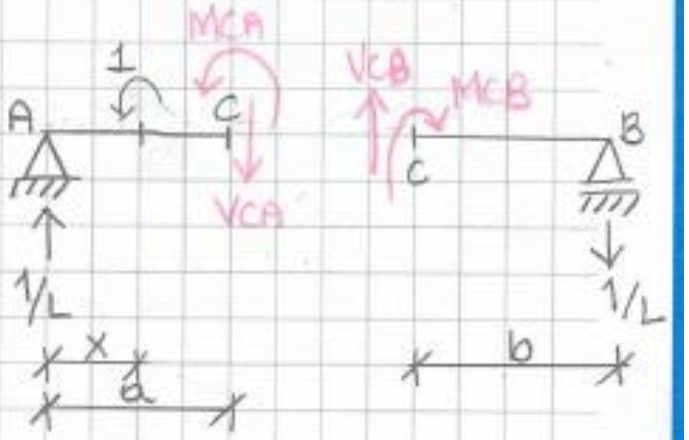
$V_B = -\frac{1}{L}$





* C noktasındaki kesme kuvveti ve eğilme momenti tesir çizgisi :

Yük A-C arasında $(0 \le x \le a)$

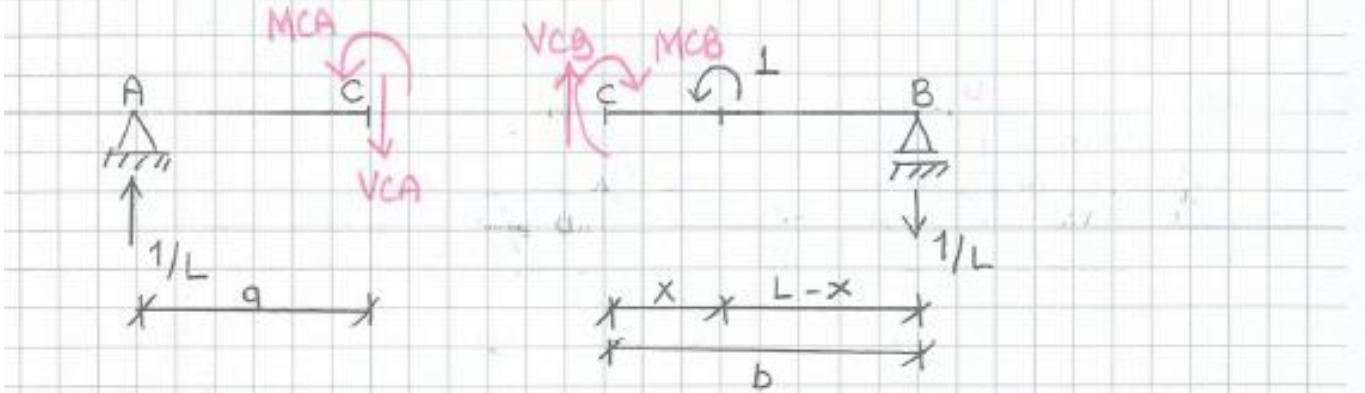


Daha az kuvvet bulunan sağ parçaya göre denklemler yazılırsa

$\Sigma Y = 0 \quad V_{CB} = \frac{1}{L}$

$\curvearrowright \Sigma M_C = 0 \quad M_{CB} = -\frac{1}{L} \cdot b$

Yük B-C arasında $a \le x \le L$

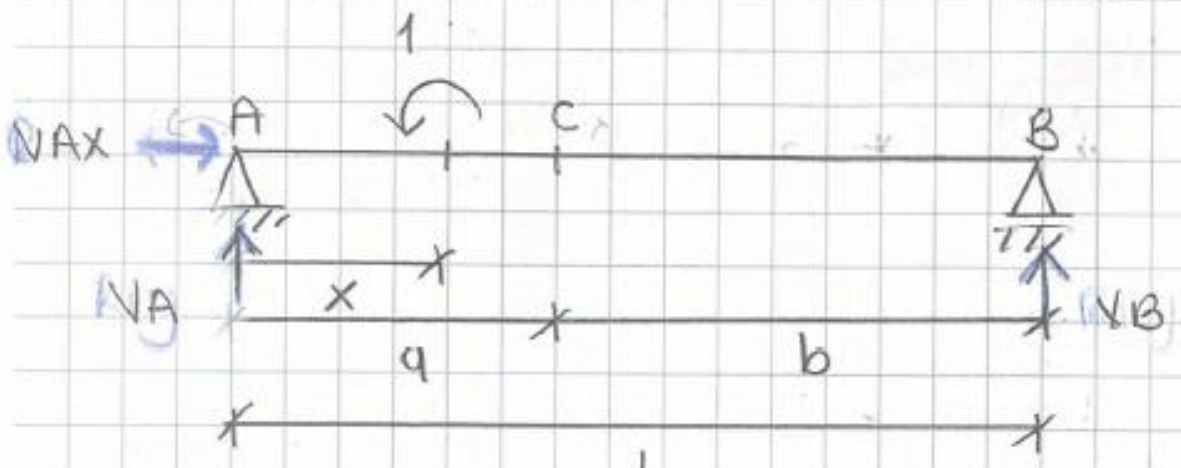


Daha az kuvvet bulunan solda kalan sistem için (A-C sistemi)

$\Sigma Y = 0 \quad V_{CA} = \frac{1}{L}$

$\curvearrowright \Sigma M_C = 0 \quad M_{CA} = \frac{1}{L} \cdot a \rightarrow M_{CA} = \frac{a}{L}$

1 Birimlik moment etkimesi durumunda;



Tüm sistemden $\Sigma X = 0$

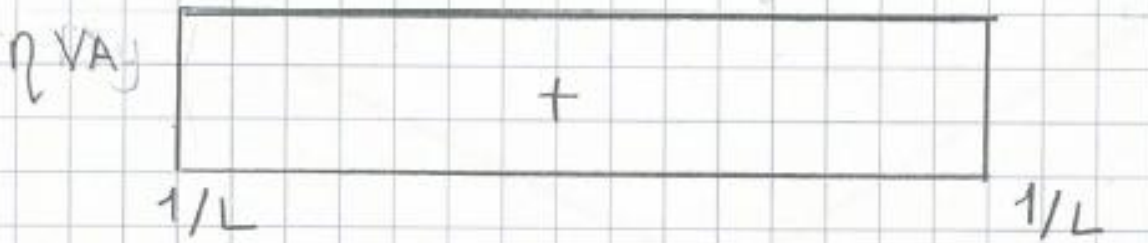
$NAX = 0$

$\Sigma M_B = 0 \quad 1 = V_A \cdot L$

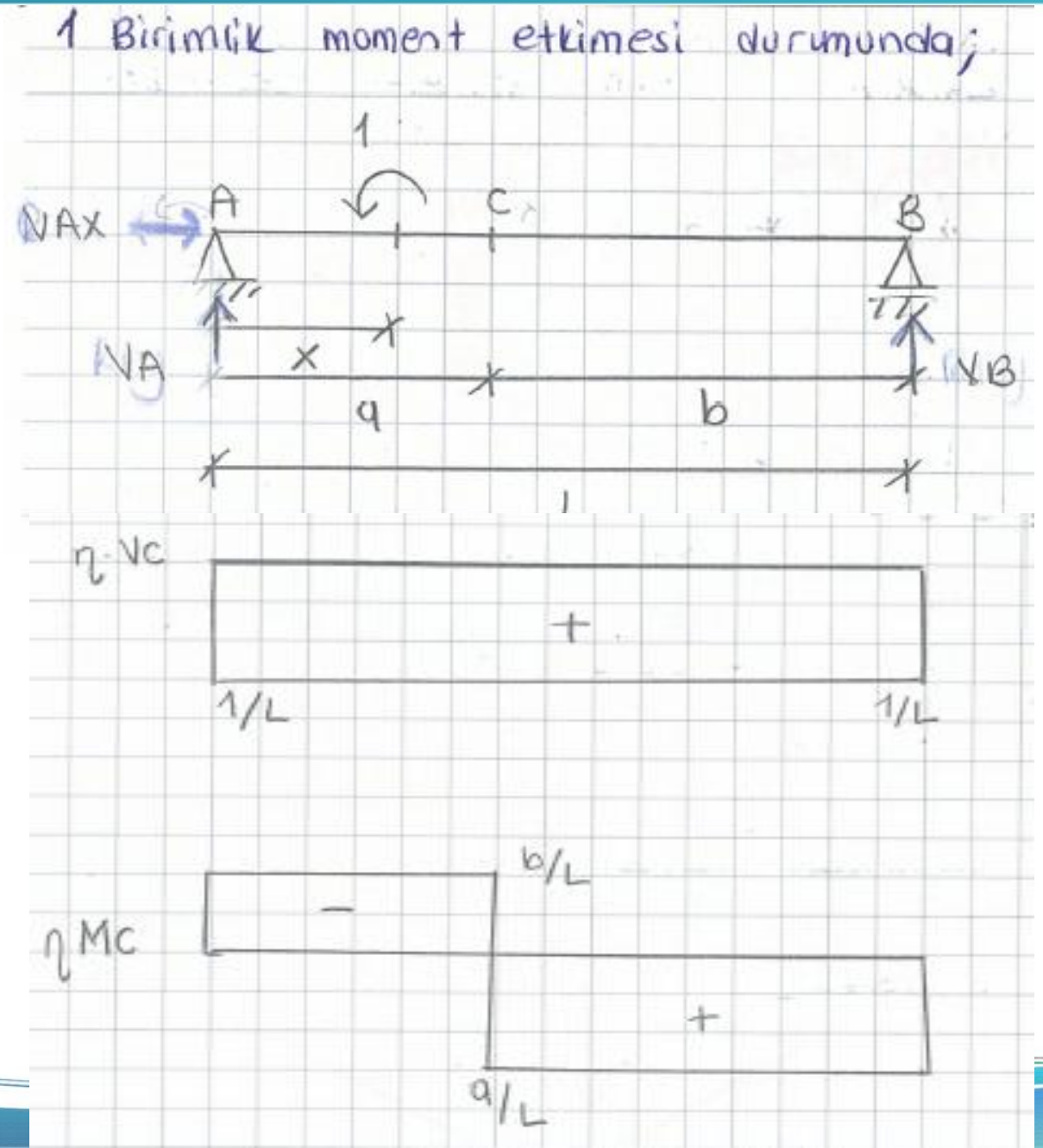
$V_A = \frac{1}{L}$

$\Sigma M_A = 0 \quad 1 + V_B L = 0$

$V_B = -\frac{1}{L}$





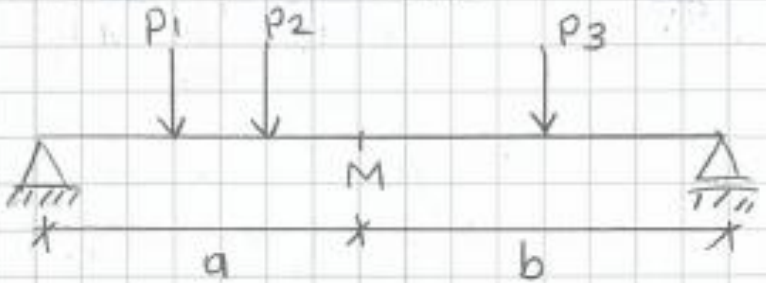

1 Birimlik moment etkimesi durumunda;
1
NAX
A
C
B
NA
x
a
b
l
NB
η·Vc
+
1/L
1/L
b/L
−
ηMc
+
a/L

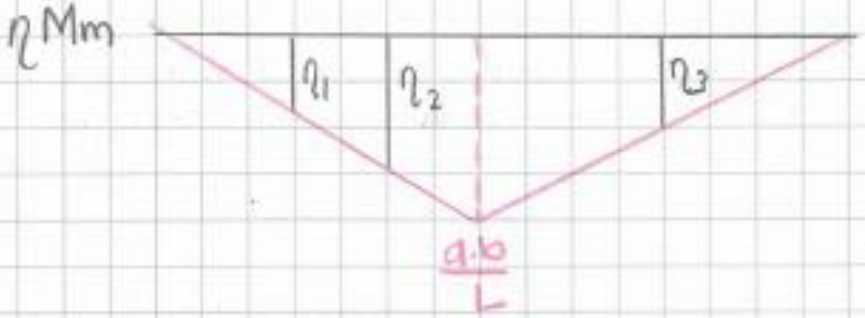
Tesir Çizgilerinin Kullanılması

Tesir çizgileri iki amaç için kullanılmaktadır.

1- Verilmiş bir kesitte sabit yüklerden meydana gelen bir kesit tesirinin hesabı
2- Verilmiş bir kesitte bir kesit tesirinin max ve min yapan katar durumun hesabı

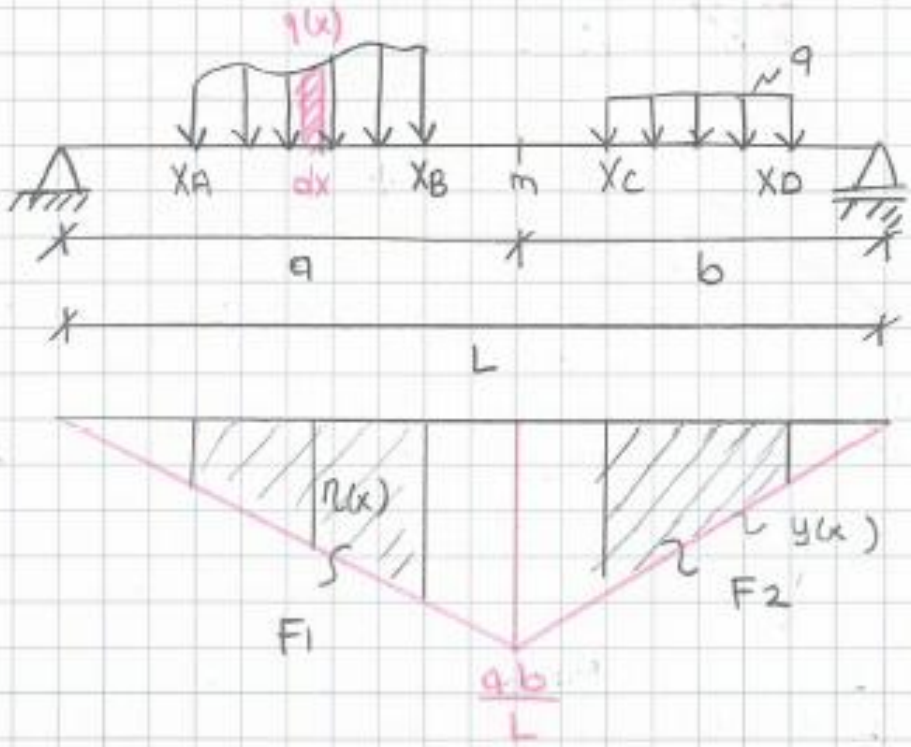
* Tekil yükler durumunda





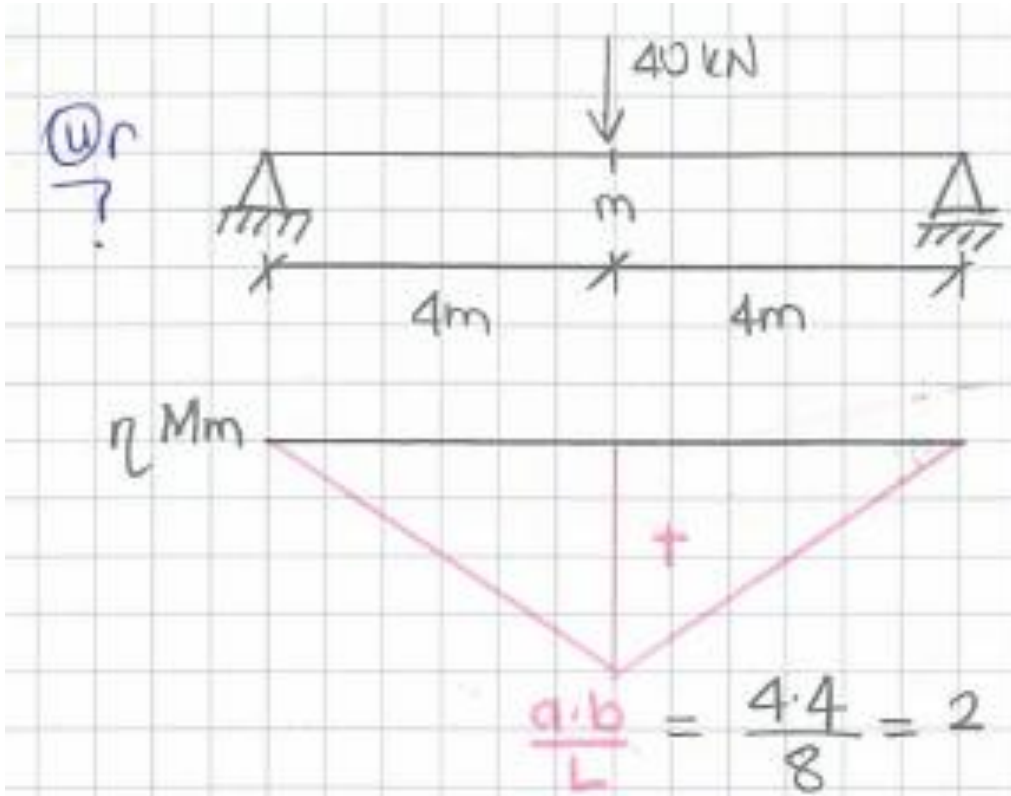
$$M_m = P_1 \cdot \eta_1 + P_2 \cdot \eta_2 + P_3 \cdot \eta_3$$

* Yayılı yük durumunda



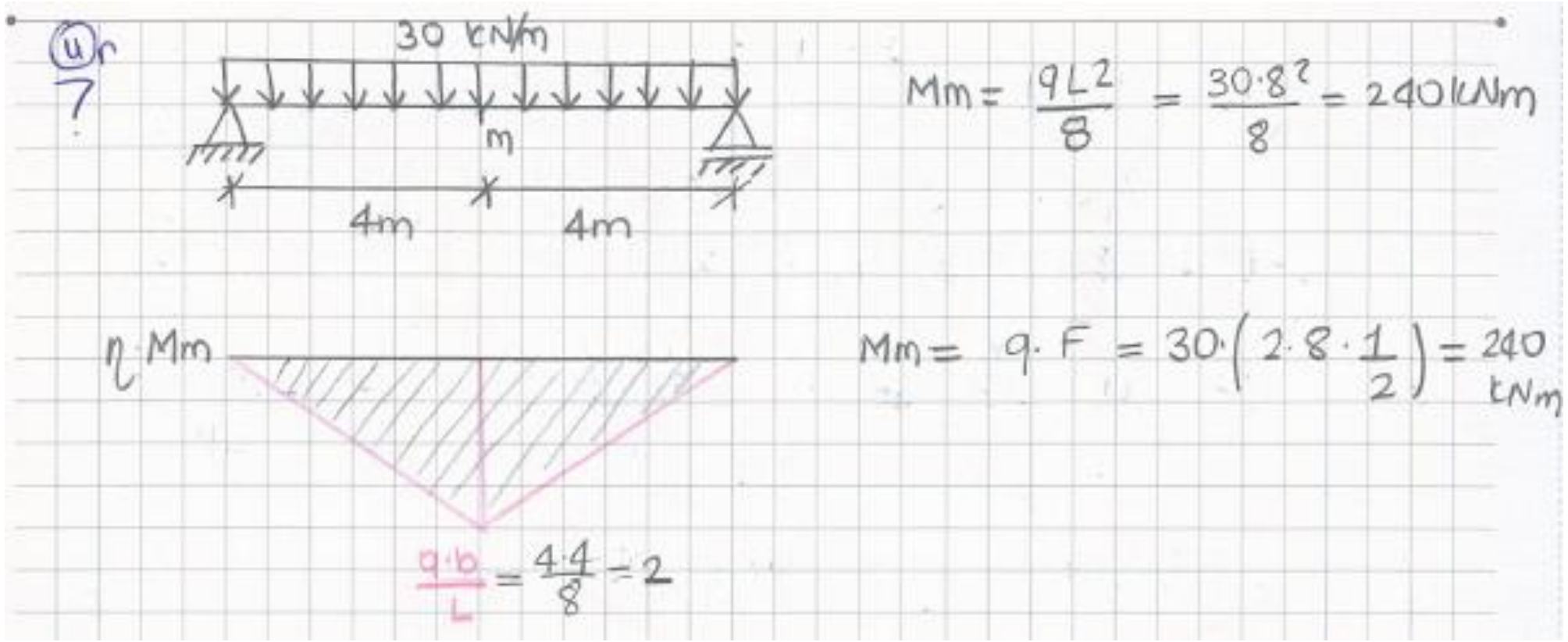
$$Mm = \int_{XA}^{XB} q(x) \cdot dx \cdot \eta(x) + \int_{XC}^{XD} q \cdot y(x)\, dx$$

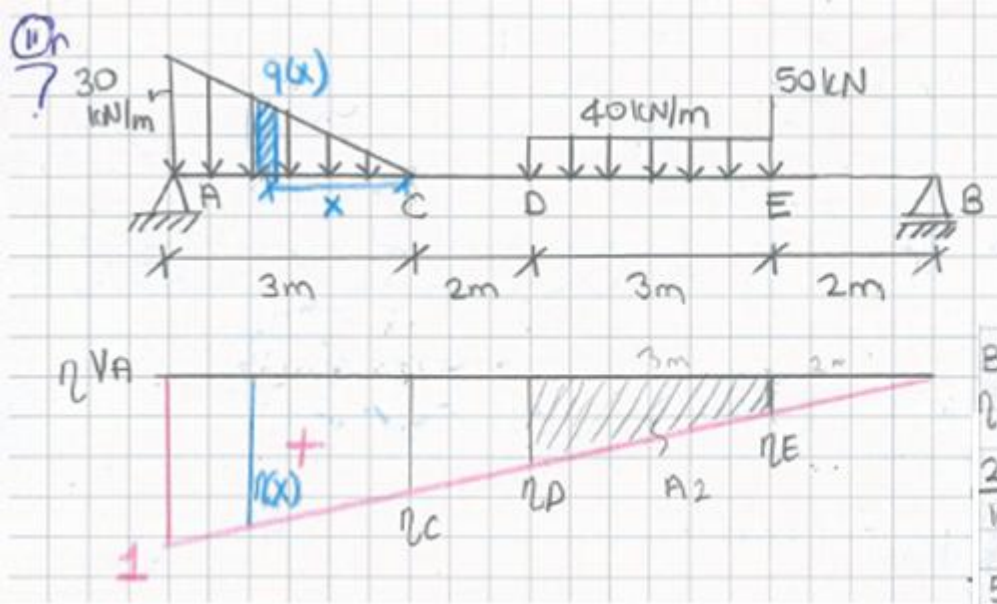
$$Mm = F1 \cdot \int_{XA}^{XB} q(x)\, dx + q \cdot F2$$



$$M_m = \frac{P \cdot L}{4} = \frac{40 \cdot 8}{4} = 80 \text{ kNm}$$

$$M_m = \eta \cdot P = 2 \cdot 40 = 80 \text{ kNm}$$


(a)
7
30 kN/m
m
4m
4m
η Mm
$\frac{q \cdot b}{L} = \frac{4 \cdot 4}{8} = 2$
$Mm = \frac{qL^2}{8} = \frac{30 \cdot 8^2}{8} = 240\ kNm$
$Mm = q \cdot F = 30 \cdot \left(2 \cdot 8 \cdot \frac{1}{2}\right) = 240\ kNm$



VA, VB, mesnet tepk. Vc, Mc kesit etki-leinni tesir cizgileinden faydalanarak hesaplayiniz.

Benzerlikten faydalanarak η_E, η_D değerleri bulunur.

$$\frac{2}{10} = \frac{\eta_E}{1} \rightarrow \eta_E = 0,2$$

$$\frac{5}{10} = \frac{\eta_D}{1} \rightarrow \eta_D = 0,5$$

$$\frac{x}{3} = \frac{q(x)}{30} \rightarrow q(x) = 10x$$

ηVA tesir cizgisi diyagramından;

$$\frac{x+7}{10} = \frac{\eta_x}{1} \rightarrow \eta_x = 0,7 + 0,1x$$

$$A_2 = \left(\frac{\eta_E + \eta_D}{2}\right) \cdot 3 = \left(\frac{0,2 + 0,5}{2}\right) \cdot 3 = 1,05\ m^2$$

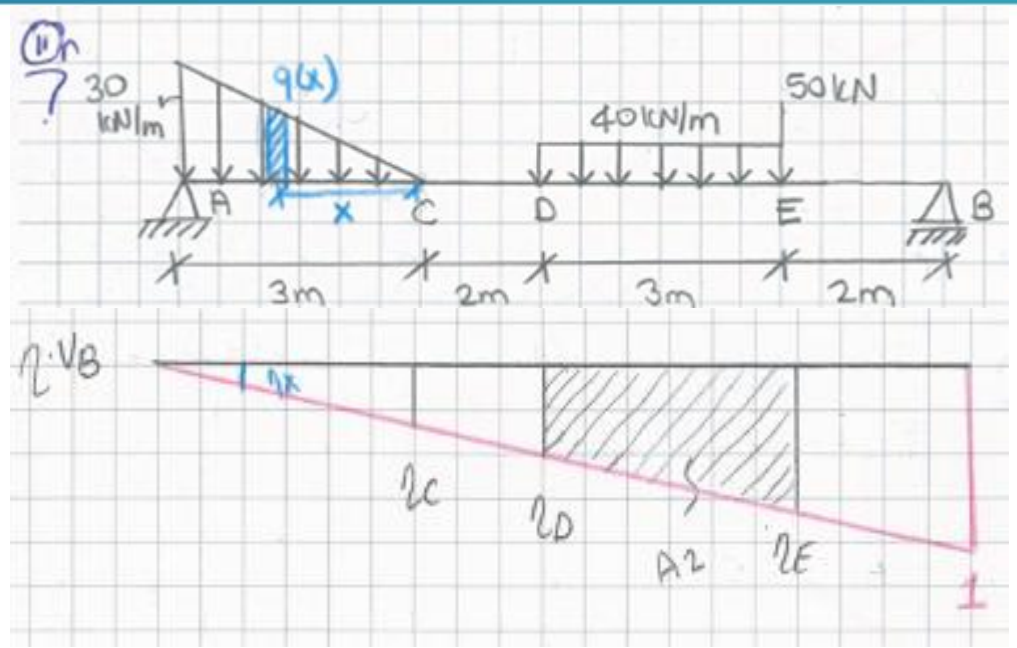
$$VA = \int_0^3 q(x) \cdot \eta_x \cdot dx + q \cdot A_2 + \eta_E \cdot P$$

$$VA = \int_0^3 10x \cdot (0,7 + 0,1x) \cdot dx + 40 \cdot 1,05 + 0,2 \cdot 50$$

$$VA = \int_0^3 (7x + x^2)\, dx + 42 + 10$$

$$VA = \frac{7x^2}{2} + \frac{x3}{3}\Big|_0^3 + 52$$

$$VA = 7 \cdot \frac{3^2}{2} + \frac{3^3}{3} + 52 \Rightarrow VA = 92,5\ kN$$



* VB mesnet tepkisini hesaplamak için

$$\frac{x}{3} = \frac{q(x)}{30} \rightarrow q(x) = 10x$$

$$\frac{3-x}{10} = \frac{\eta x}{1} \rightarrow \eta x = 0{,}3 - 0{,}1x$$

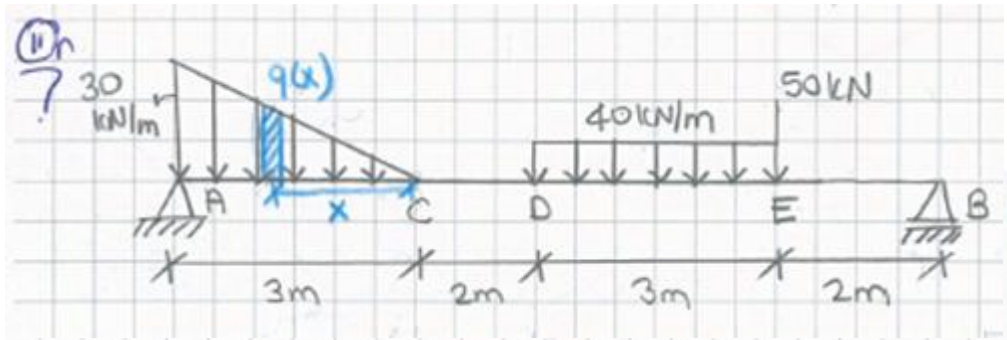
$$A_2 = \left(\frac{0{,}5 + 0{,}8}{2}\right) \cdot 3 \rightarrow A_2 = 1{,}95 \ m^2$$

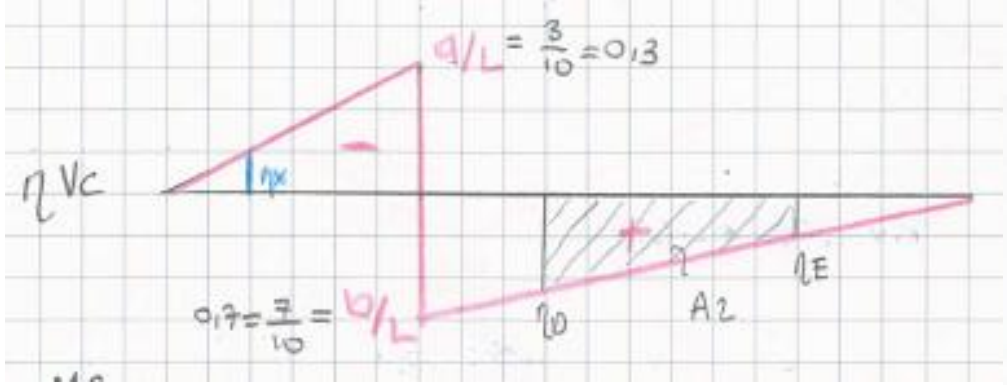
$$V_B = \int_0^3 q(x) \cdot \eta x \cdot dx + q \cdot A_2 + P \cdot \eta E$$

$$V_B = \int_0^3 10x \cdot (0{,}3 - 0{,}1x) \cdot dx + 40 \cdot 1{,}95 + 50 \cdot 0{,}8$$

$$V_B = \left. 3\frac{x^2}{2} - \frac{x^3}{3} \right|_0^3 + 78 + 40$$

$$V_B = 3 \cdot \frac{3^2}{2} - \frac{3^3}{3} + 118 \rightarrow V_B = 122{,}5 \ kN$$





✱ Vc Kesme Kuvvetinin bulunması:

η'lar alınırken işaretleri dikkat alınır.

$$\frac{\eta_x}{3} = \frac{3-x}{3} \rightarrow \eta_x = 0,3 - 0,1x \quad , \quad q(x) = 10x$$

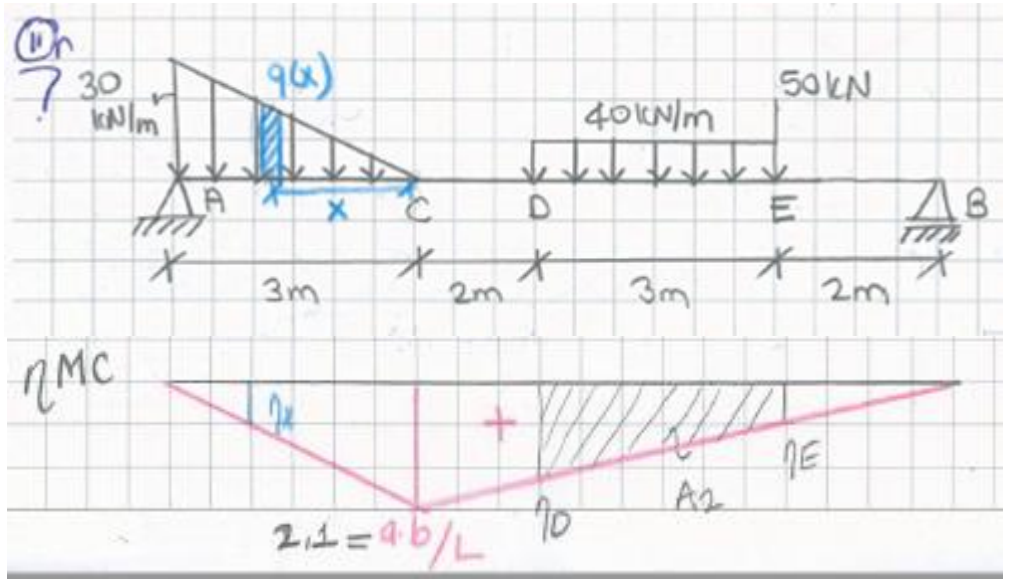
$$\frac{\eta_E}{0,7} = \frac{1}{7} \rightarrow \eta_E = 0,2$$

$$\frac{\eta_D}{0,7} = \frac{5}{7} \rightarrow \eta_D = 0,5$$

$$V_C = \int_0^3 q(x) \cdot \eta_x \cdot dx + q \cdot A_2 + P \cdot \eta_E$$

$$V_C = \int_0^3 \cdot 10x \cdot (0,3 - 0,1x)\, dx + 40 \cdot \left(\frac{0,2+0,5}{2}\right) \cdot 3 + 50 \cdot 0,2$$

$$V_C = \frac{x^3}{3} - \frac{3x^2}{2}\Big|_0^3 + 42 + 10 \Rightarrow V_C = 47,5 \text{ kN}$$



✱ Mc eğilme momentinin bulunması :

$$\frac{\eta_x}{2,1} = \frac{3-x}{3} \rightarrow \eta_x = 2,1 - 0,7x$$

$$M_C = \int_0^3 q(x)\cdot\eta_x\cdot dx + q\cdot A_2 + P\cdot\eta_E$$

$$M_C = \int_0^3 10x\cdot(2,1-0,7x)\,dx + 40\cdot\left(\frac{0,6+1,5}{2}\right)3 + 50\cdot 0,6$$

$$M_C = 21\frac{x^2}{2} - 7\frac{x^3}{3}\Big|_0^3 + 126 + 30 \Rightarrow M_C = 187,5\ \text{kNm}$$

KAYNAKLAR

1. **Ekiz, İ. Yapı Statiği I, Seç yayın Dağıtım, İstanbul, 1995.**
2. **KTU Yapı Statiği I ders notları**
3. **K. Girgin, M.G. Aksoylu, Y. Durgun, K. Darılmaz, Yapı Statiği, İzostatik Sistemler, Çözümlü Problemler, Birsen Yayınevi, İstanbul, 2011**
4. **İTÜ Yapı Statiği I ders notları**
5. Karadoğan F, Pala S, Yüksel E, Durgun Y, Yapı Mühendisliğine Giriş, Yapısal Çözümleme, 2011, No. 1, Vol. 1, İstanbul: Birsen Yayınevi
6. Çakiroglu, A., Çetmeli, E. Yapı statiği I, Beta Basım Yayım Dağıtım A.S., İstanbul, 1990.